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CHARACTERIZATION AND MODELING OF THE  
DIGITAL HIGH-SPEED AUTOVON CHANNEL -  
INTERIM REPORT

AUGUST 1975

Prepared for

DEPUTY FOR COMMAND AND MANAGEMENT SYSTEMS  
ELECTRONIC SYSTEMS DIVISION  
AIR FORCE SYSTEMS COMMAND  
UNITED STATES AIR FORCE  
Hanscom Air Force Base, Bedford, Massachusetts



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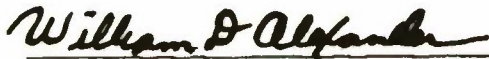
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is evaluated in terms of the residual channel error distributions associated with the use of the state-of-the-art Codex 9600 modem. It is demonstrated that the channel is a burst error channel, will support data transmission with bit error rates in the order of  $10^{-4}$  to  $10^{-5}$ , and can be modeled by a MARKOV transition matrix.

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## FOREWORD

The research reported in this paper was performed by the MITRE Corporation in conjunction with the Digital Communications Experimental Facility, DICEF, of the Rome Air Development Center (RADC), Griffiss, AFB. Channel tests of AUTOVON were conducted by RADC and the digital data was analyzed at MITRE. Special appreciation must be given to Mr. J. McEvoy of RADC/DCLD who helped organize the tests and placed many of the test requirements in proper perspective and to Capt. C. Lownes, RADC/DCLD, who was test director.

The data analysis provided in this paper covers only that portion of the data in the possession of MITRE at the time the paper was prepared. Since this constitutes only one-half the total data, care should be used in drawing any conclusions. The data reported on herein was itself received in halves with the two halves being radically different in nature.

The Codex 9600 modem was used solely because it was readily available at RADC. This study was in no way a modem evaluation.

The AUTOVON switches used for creating tandem links were selected solely because dial-thru units were available at these switches at the time the tests started.



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## SECTION I

### INTRODUCTION

In recent years there has been within both the civilian and military worlds a drive toward the use of voice grade circuits (nominally 2400 Hz of useable bandwidth) for high speed digital data transmission. This drive has developed because of the ever increasing use of computers and computer-like devices and the desire to connect on existing communication circuits such devices at data rates sufficiently high for computer efficiency.

A natural outgrowth of efforts in this direction is the use of the AUTOVON common-user voice grade circuits for digital data transmission at the state-of-the-art speed of 9600 b/s.

The Electronic Systems Division of the Air Force, is presently developing with MITRE's technical support a new CONUS record data communication system for SAC called the SAC Automated Total Information Network (SATIN IV). Since AUTOVON is the primary candidate for use as the backbone transmission facility for SATIN IV, a study was undertaken to determine the digital characteristics of AUTOVON in light of the SATIN IV system performance requirements. As a first step in this study the AUTOVON digital error patterns are being analyzed. The ultimate use of the error pattern data will be an investigation of whether-or-not SAC-SATIN IV performance requirements for error control and response time can be met on AUTOVON.

An interim description of the AUTOVON channel is provided, using an analysis technique previously developed (1) for such error pattern tests. A mathematical channel model also has been developed. This model provides a convenient method to reproduce the statistics of the error pattern data obtained.

#### AUTOVON

AUTOVON is basically a leased polygrid of telephone circuits which traverse cable and microwave links criss-crossing the country in the same fashion as the commercial telephone system. The polygrid contains switches (ESS and CROSSBAR) of the same type as commercial communications. These switches perform the call routing and inter-connection functions of AUTOVON. The polygrid is made up of Common Grade Leased Lines, which while not conditioned, nominally meet C2 specifications and access lines which are conditioned C3 (referred to as S3 by the government). A brief summary of the nominal conditioning levels is given in Table 1.

TABLE 1  
BELL SYSTEM CIRCUIT PARAMETERS

Frequency Range	C2 Amplitude Variation (dB)	C3 Amplitude Variation (dB)
0.3 - 3.0 kHz	-2 to +6	-0.8 to +2
0.5 - 2.8 kHz	-2 to +3	-0.5 to +1
Envelope Delay Distortion ( $\mu$ SEC)		
0.5 - 2.8 kHz	3000	650
0.6 - 2.6 kHz	1500	300
1.0 - 2.6 kHz	500	110

The nominal conditioning levels while useful in evaluating the performance of analog systems are not generally relevant to digital data transmission performance. Such performance is more closely related to channel noise and phase jitter and how the decision algorithm of the modem responds to these channel conditions. Thus the true digital data channel includes both AUTOVON and the modem used. The modem chosen for the tests was the Codex 9600 modem. This modem was chosen solely because it was the only 9600 b/s telephone line data modem on-hand and owned by the U. S. Government at the RADC test site at the time the tests were initiated.

#### The Codex 9600 Modem

The Codex 9600 modem is designed to transmit 4800, 7200 or 9600 bps serial, synchronous digital data at a 2400 baud signaling rate

over a dedicated type 3002, C2 conditioned 4-wire telephone circuit. It is a full duplex, double sideband suppressed carrier modem using a combination of amplitude- and phase-shift keying and transversal equalization. The transmitted signal occupies a 2400 Hz spectrum centered at 1706 Hz. Each baud contains information from a 4-bit sample of 9600 bps, a 3-bit sample of 7200 bps, or a 2-bit sample of 4800 bps input data. Input data is scrambled before transmission to prevent the receiver from becoming sensitive to data patterns and to provide a uniform line-signal spectrum for the equalization process. Receiver carrier and timing recovery circuits use information contained in the transmitted data to eliminate the need for the transmission of pilot tones.

The modem employs a digital adaptive equalizer which is the digital equivalent of a tapped transversal delay line filter without feedback. The equalizer performs a complex valued digital filtering operation on the most recent thirty-one samples of the in-phase and quadrature components of the received signal and provides, at each baud time, a pair of outputs which correspond to the in-phase and quadrature components of the equalized received signal. The data decision logic also generates in-phase and quadrature error signals representing the difference between the equalizer output signals and the selected data point. These error signals are fed back to the equalizer and used to update the tap coefficients.



The magnitude of the error signal from the data decision logic also provides an indication of the reliability of the data. When the error signal reaches a pre-set point, the modem initiates a training mode to provide equalizer settings beyond the normal adaptive range. During retraining a known sequence is transmitted and the equalizer adjusts its taps by using knowledge of the transmitted sequence during this period rather than making decisions on the received data. The training mode takes approximately 280 milliseconds during which the output of the modem is either all ones or zeros depending on how it was strapped.

#### Test Procedure

The AUTOVON performance was measured by establishing a communication facility at the RADC-DICEF and transmitting through the transmit modem to a selected AUTOVON switch. At the switch an automatic re-dialing unit established a link with either another switch or the RADC-DICEF facility where the receive modems were located. The test procedure was to first establish the link or series of links and then transmit a known bit pattern through the modem. When the signal was received by the receive modem and its decisions as to bit values were made the received bit sequence (suitably delayed to account for transmission delay) was added modulo 2 with no carry to the transmitted sequence. This summation (a bit-by-bit error pattern) was then recorded on computer compatible magnetic tape in a suitable format for later statistical analysis. While dialing was to target switches

the trunks were randomly selected by the inherit nature of call dialing.

In all cases data transmission originated at RADC and proceeded via access line to the Tully, N. Y. AUTOVON switch. From Tully connections were made to the switches at Pottstown, Pa., Arlington, Va., Rockdale, Ga., and Santa Rosa, Ca. in varying orders and combinations. The return connection was back to RADC via Tully. Of the switches used only Arlington, Va. was an ESS. Testing has been conducted at all times of the day and test runs have for the most part been of 30 minutes or 1 hour duration with re-dialing between runs. The re-training mode of the Codex modem was disabled, during the course of the test calls.

#### Summary of Data

Data was collected at 4800 b/s and 9600 b/s on combinations of AUTOVON links. The data to be considered is summarized in Table 2. Attempts were made to obtain equal amounts of data. For the most part the sample size inequality in the 4800 b/s data is due solely to the interim nature of this paper. The data sample sizes should be equalized when the testing is completed. In the case of the 9600 b/s data the story is different. It has been totally impossible to tandem five AUTOVON links at 9600 b/s. The error rate encountered was too high (50%) to allow the test equipment to maintain synchronization. Similarly, there have been difficulties in obtaining useful

TABLE 2  
DATA SUMMARY

Selected Switch Connectivity*	Data Rate	Total Bits	Total Errors	Bit Error Rates
Two Switches	4800 b/s	1,257,078,507	32657	$2.6 \times 10^{-5}$
Three Switches	4800 b/s	534,428,237	25035	$4.7 \times 10^{-5}$
Four Switches	4800 b/s	321,458,175	34277	$1.07 \times 10^{-4}$
Five Switches	4800 b/s	131,585,800	5878	$4.5 \times 10^{-5}$
Two Switches	9600 b/s	1,322,319,493	80958	$6.1 \times 10^{-5}$
Three Switches	9600 b/s	100,240,103	13551	$1.35 \times 10^{-4}$
Four Switches	9600 b/s	16,997,353	2931	$1.41 \times 10^{-4}$
*Tully switch is used twice on each connection but counted once in identifying unique switches.				

data from three and four link tandem connections.\*\*

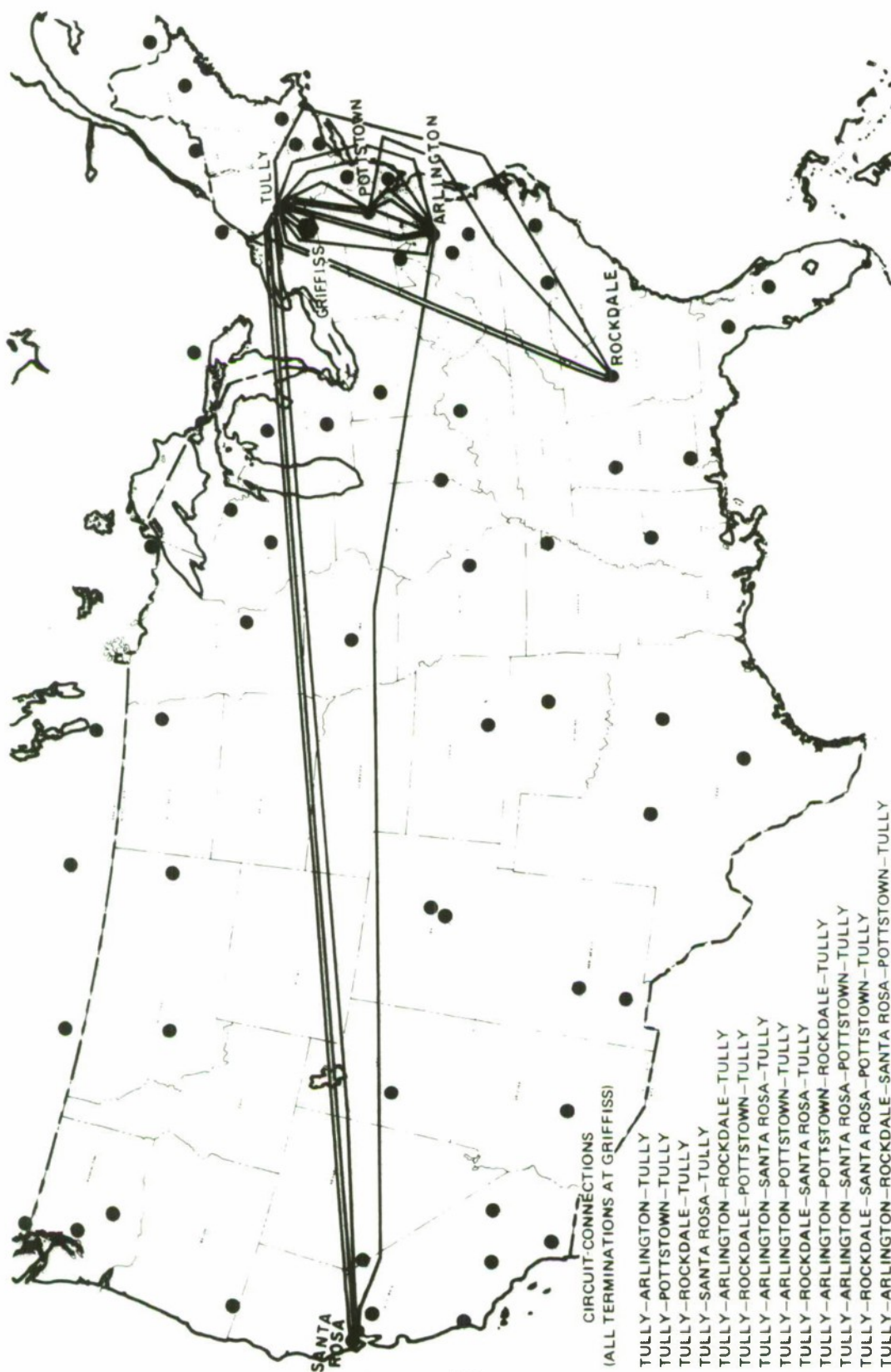
The data reported on herein represents one or two days per week of testing from August through December 1973. The connections of switches used at 4800 b/s and 9600 b/s are depicted on Maps 1 and 2 respectively.\*\*\* Since it is not now known which AUTOVON circuit will be used by SATIN IV the data cannot be directly related to SATIN IV. It is expected to be worst case since it was primarily taken on the heavily loaded east coast circuits of AUTOVON.

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\*\*Since it takes as many as two links to reach some of the switches, some of the possible switch combinations have had a small data yield.

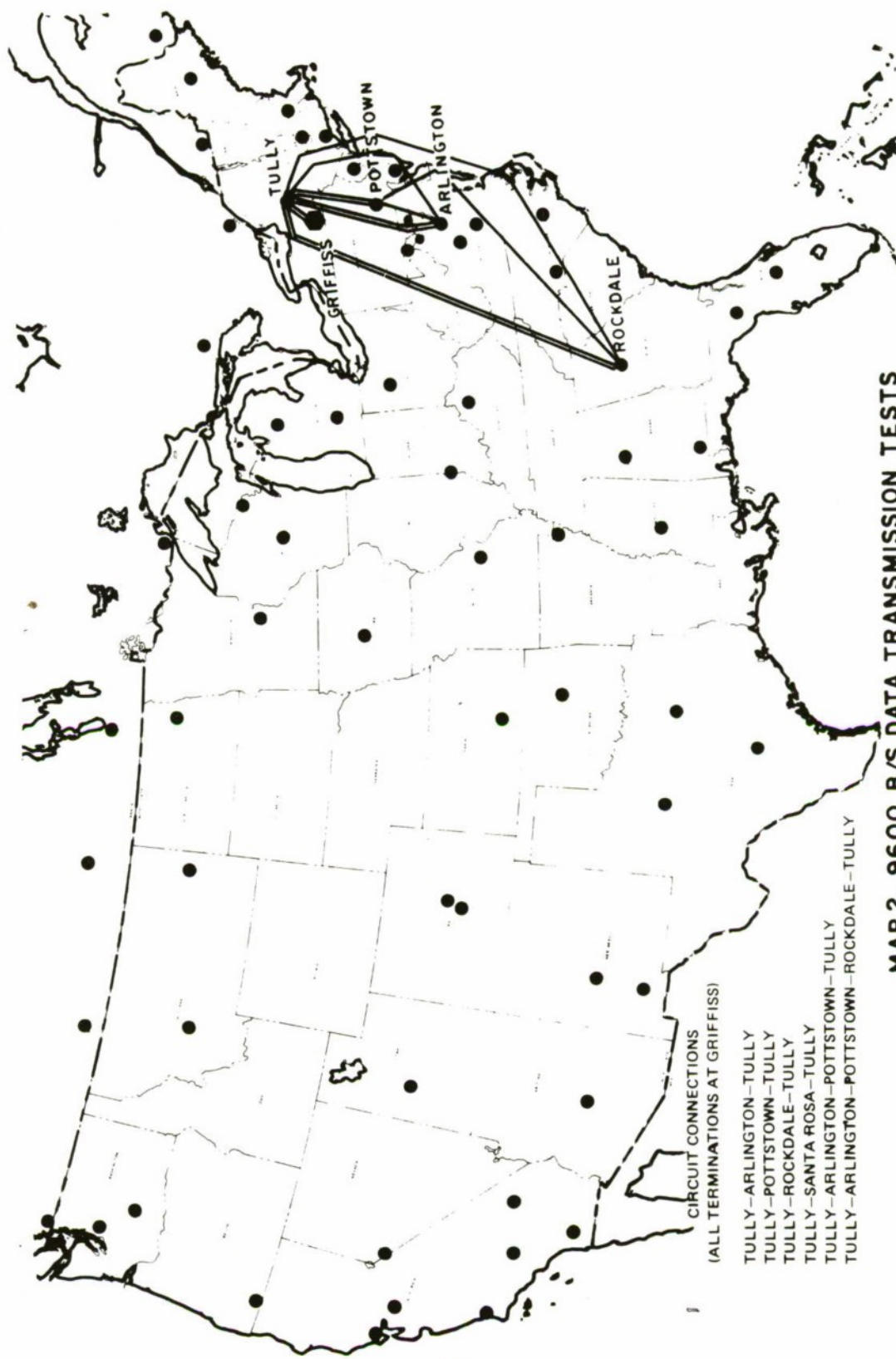
\*\*\*All connections are listed in the legends of Maps 1 and 2 but are not shown on the maps for reasons of map clarity.





MAP I. 4800 B/S DATA TRANSMISSION TESTS





MAP 2. 9600 B/S DATA TRANSMISSION TESTS

CIRCUIT CONNECTIONS  
(ALL TERMINATIONS AT GRIFFISS)

- TULLY-ARLINGTON-TULLY
- TULLY-POTTSTOWN-TULLY
- TULLY-ROCKDALE-TULLY
- TULLY-SANTA ROSA-TULLY
- TULLY-ARLINGTON-POTTSTOWN-TULLY
- TULLY-ARLINGTON-POTTSTOWN-ROCKDALE-TULLY

## SECTION II

### INTER-ERROR PROBABILITIES

In the analysis of the patterns of errors which occur in digital data transmission the most valuable probabilities are the inter-error probability distribution functions. The most important inter-error probabilities are the distribution function of consecutive errors, error occurrences ranging from one through the maximum number occurring, and the distribution function of consecutive error-free gaps between errors, lengths ranging from zero through the maximum number occurring. It is these distributions that are commonly used in channel modeling. Along with these another relevant probability is the probability of at least one error in a block of  $R$  bits. Since most practical data transmission systems transmit data in finite length blocks this probability is the probability that a message block will be received in error.

The distribution functions described above, as well as all others described in this report, were evaluated using previously developed statistical analysis programs.

#### Consecutive Error Distributions

In Figures 1 through 5 the cumulative distribution functions (expressed in percent) of consecutive errors are presented for the 4800 b/s data, the 9600 b/s data and comparisons of the two data rates for the same switch connectivities. At 4800 b/s the number

IA-42,795

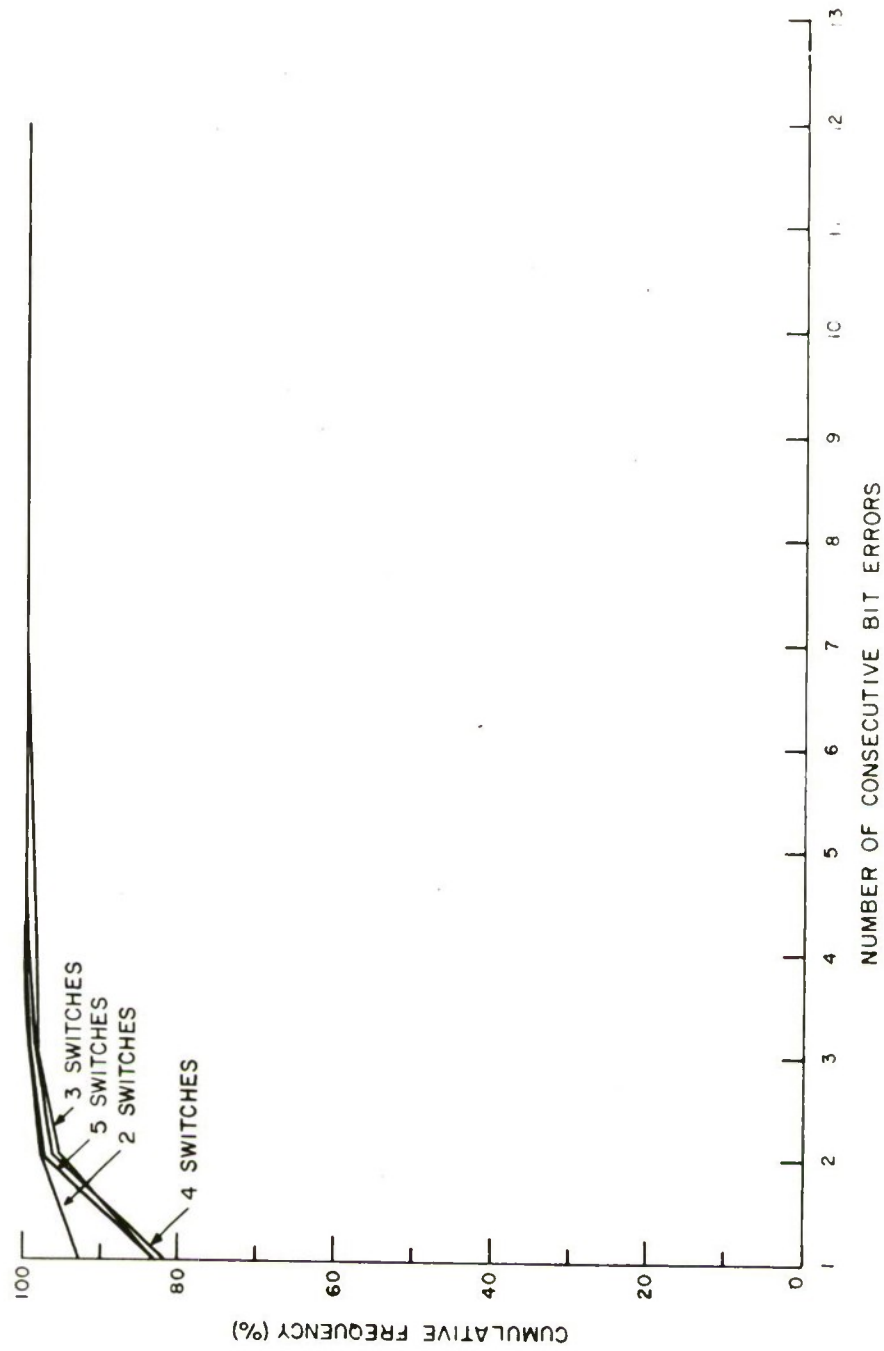


Figure 1. CUMULATIVE DISTRIBUTION OF CONSECUTIVE ERRORS — 4800 b/s

IA-42,796

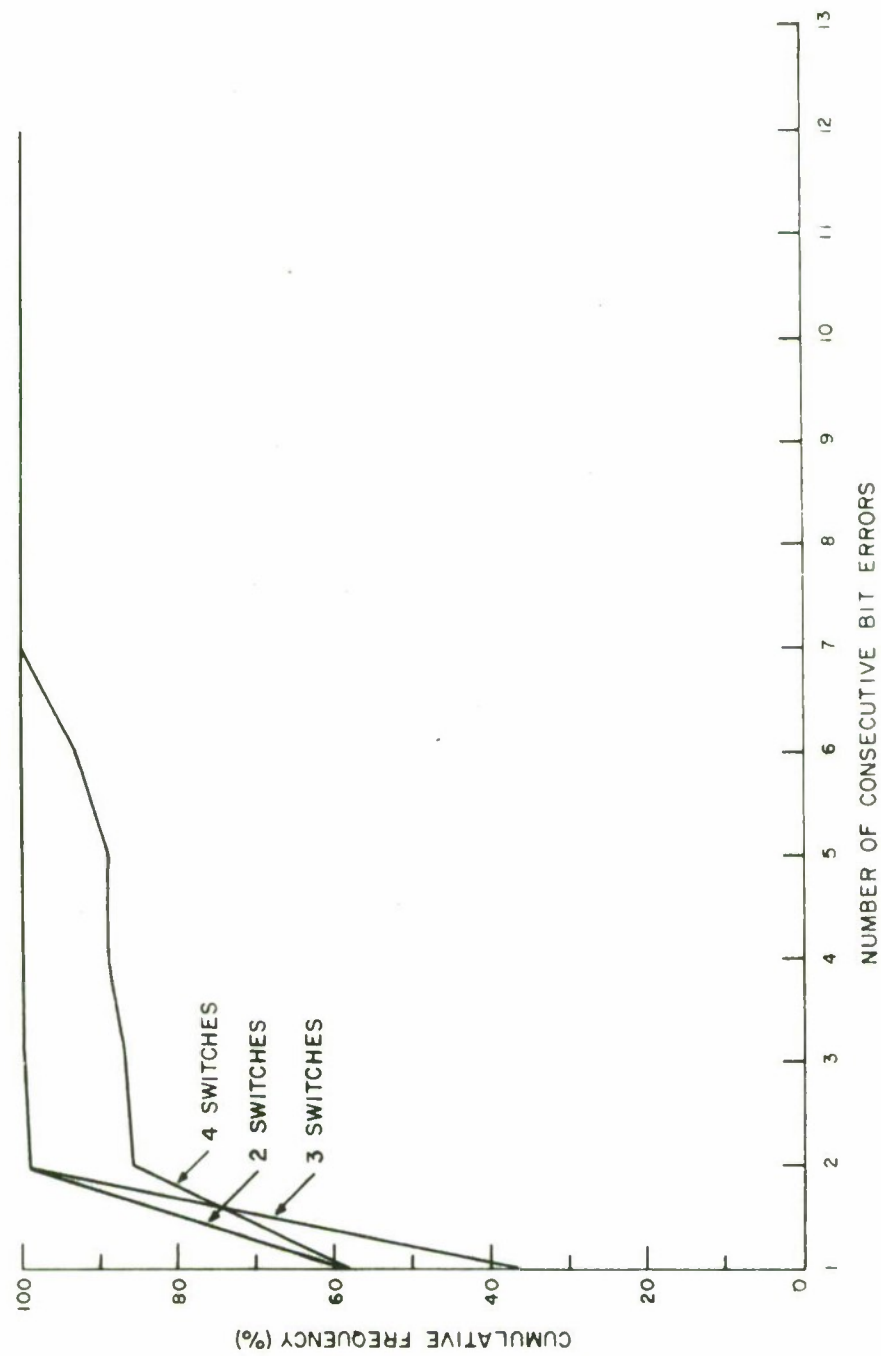


Figure 2. CUMULATIVE DISTRIBUTION OF CONSECUTIVE ERRORS — 9600 b/s

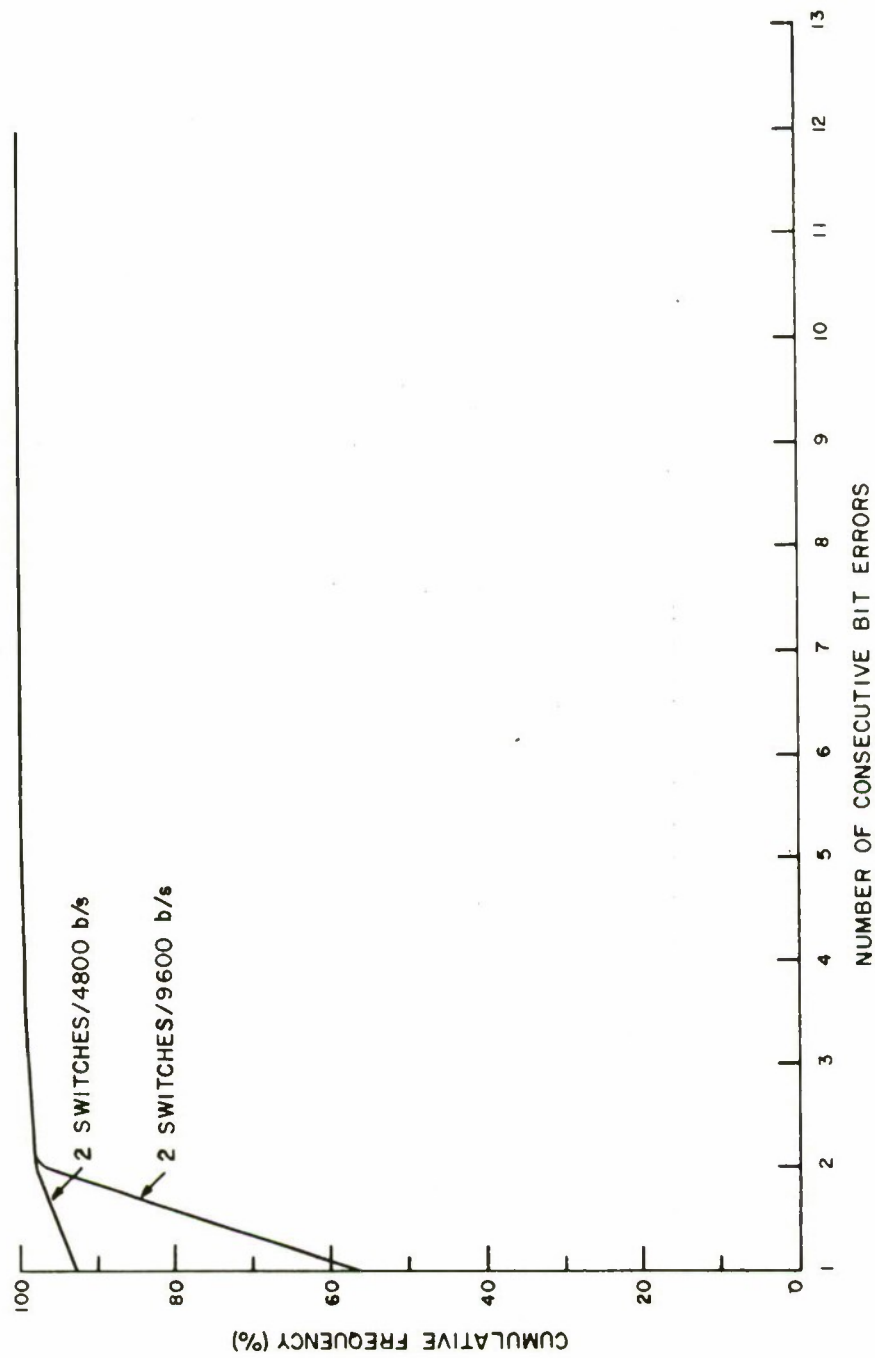


Figure 3. CUMULATIVE DISTRIBUTION OF CONSECUTIVE ERRORS —  
4800 vs 9600 b/s, 2 SWITCH COMPARISON



IA-42,798

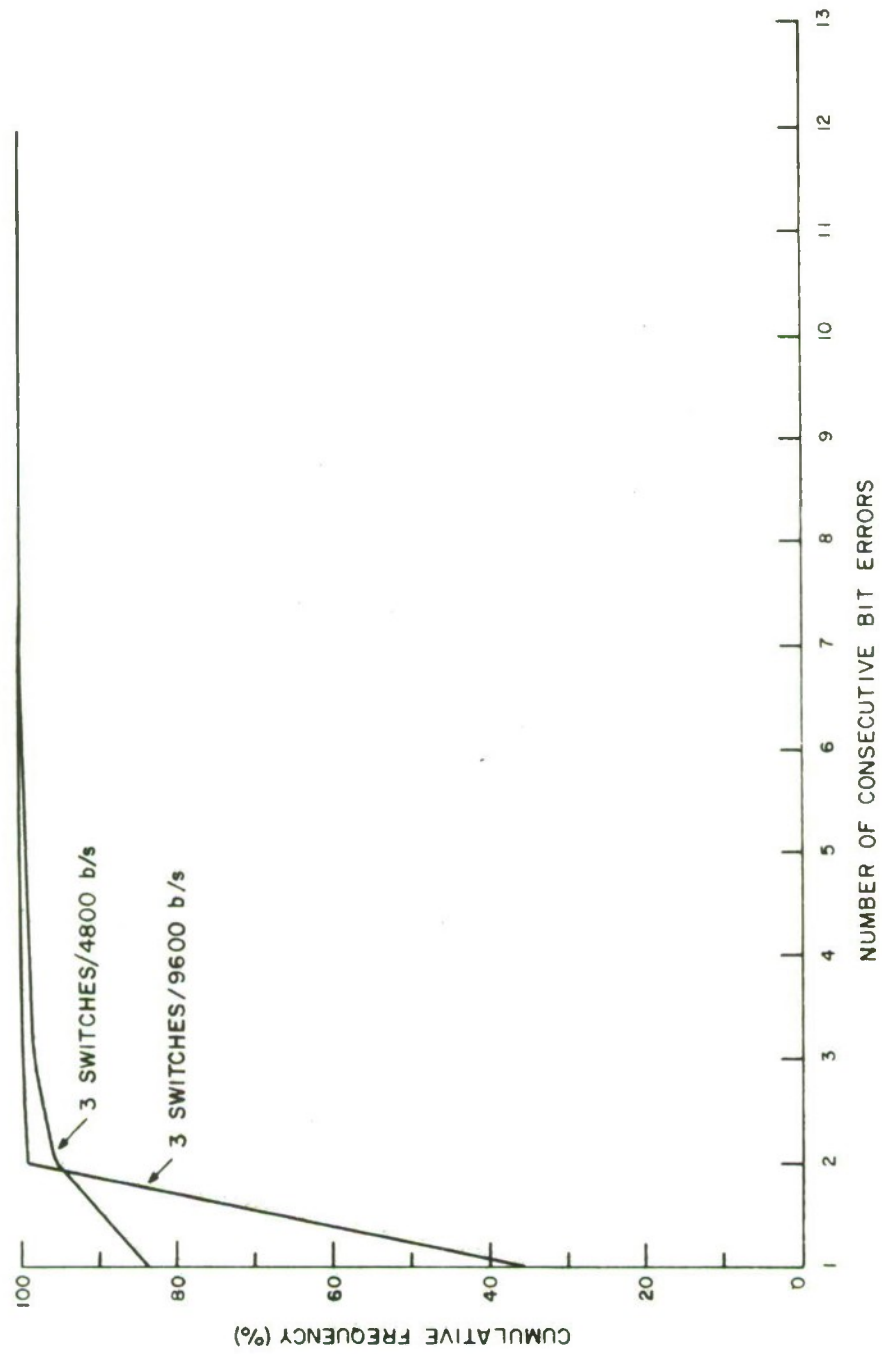


Figure 4. CUMULATIVE DISTRIBUTION OF CONSECUTIVE ERRORS —  
4800 vs 9600 b/s, 3 SWITCH COMPARISON

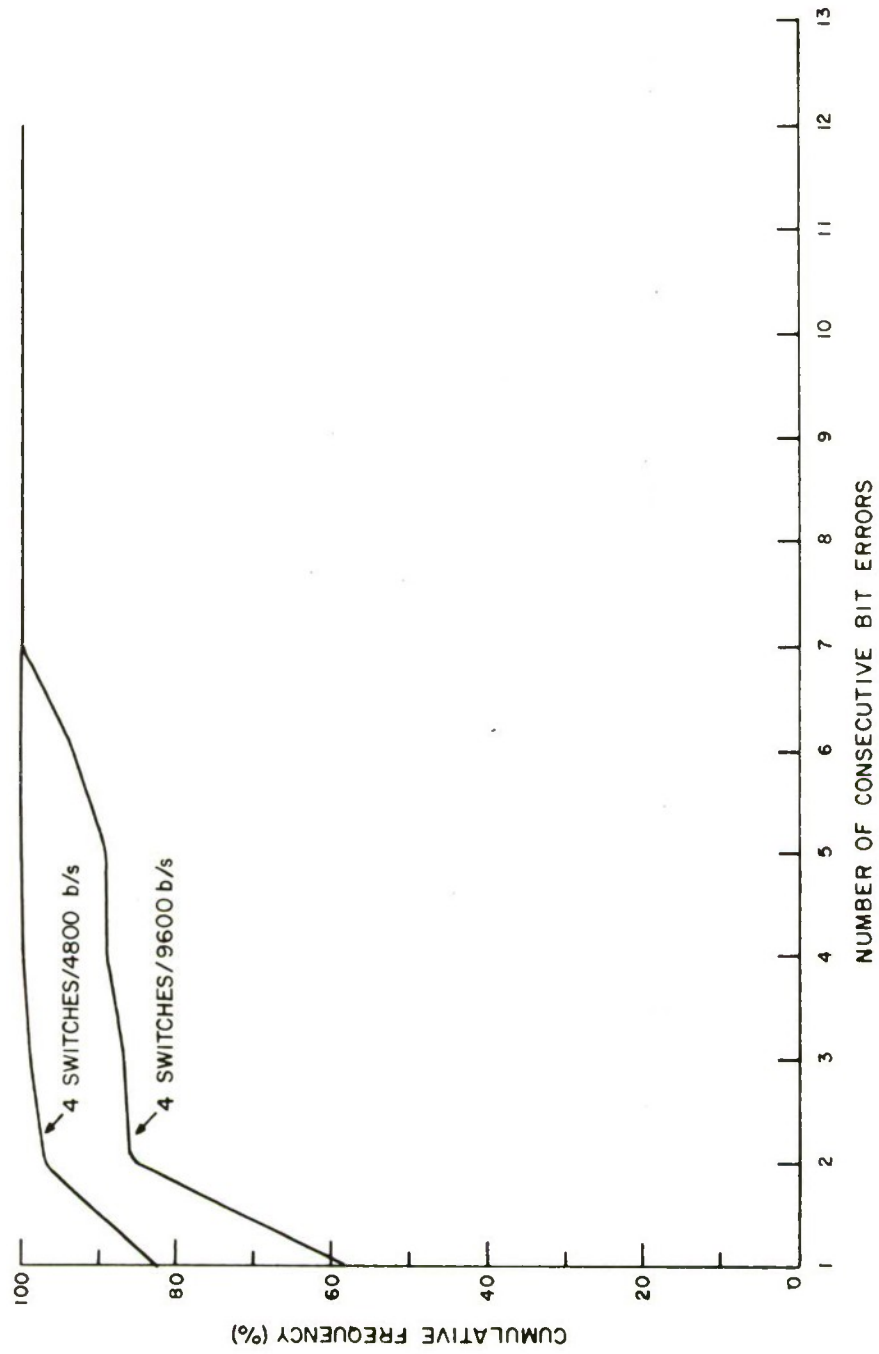


Figure 5. CUMULATIVE DISTRIBUTION OF CONSECUTIVE ERRORS —  
4800 vs 9600 b/s, 4 SWITCH COMPARISON

of consecutive bit errors is generally no greater than four although as many as 12 consecutive errors do occur. These distributions do not differ significantly with increases in the number of switches placed in tandem. At 9600 b/s the data is again essentially the same on the two switch and three switch configurations, but is radically different on the four switch tandem connection where as many as seven consecutive errors occur much more frequently than in all other samples of data.\* Also of interest is the fact that at 4800 b/s single errors predominate while at 9600 b/s double consecutive errors are most common. These double errors are caused by amplitude jumps in the received signal which in turn cause double decision errors in the Codex modem. Since, at 9600 b/s, the modem is phase differential with two-bit phase elements separated by two bit amplitude elements other possible causes of double bit errors would cause patterns of propagated errors which are not seen in the data. This absence of such patterns verifies the casual conclusion.

#### Error-free Gap Distributions

The error-free gap distributions, presented in Figures 6 through 10, were initially somewhat surprising. Telephone circuits were expected to exhibit short solid bursts of errors. These solid bursts were seen to an extent in the consecutive error distributions but the error-free gap distributions are more characteristic of the time-diffuse burst fading channel such as that found on HF radio (1).

\*Since there is little data in this group  
it is not clear whether or not this difference is significant.

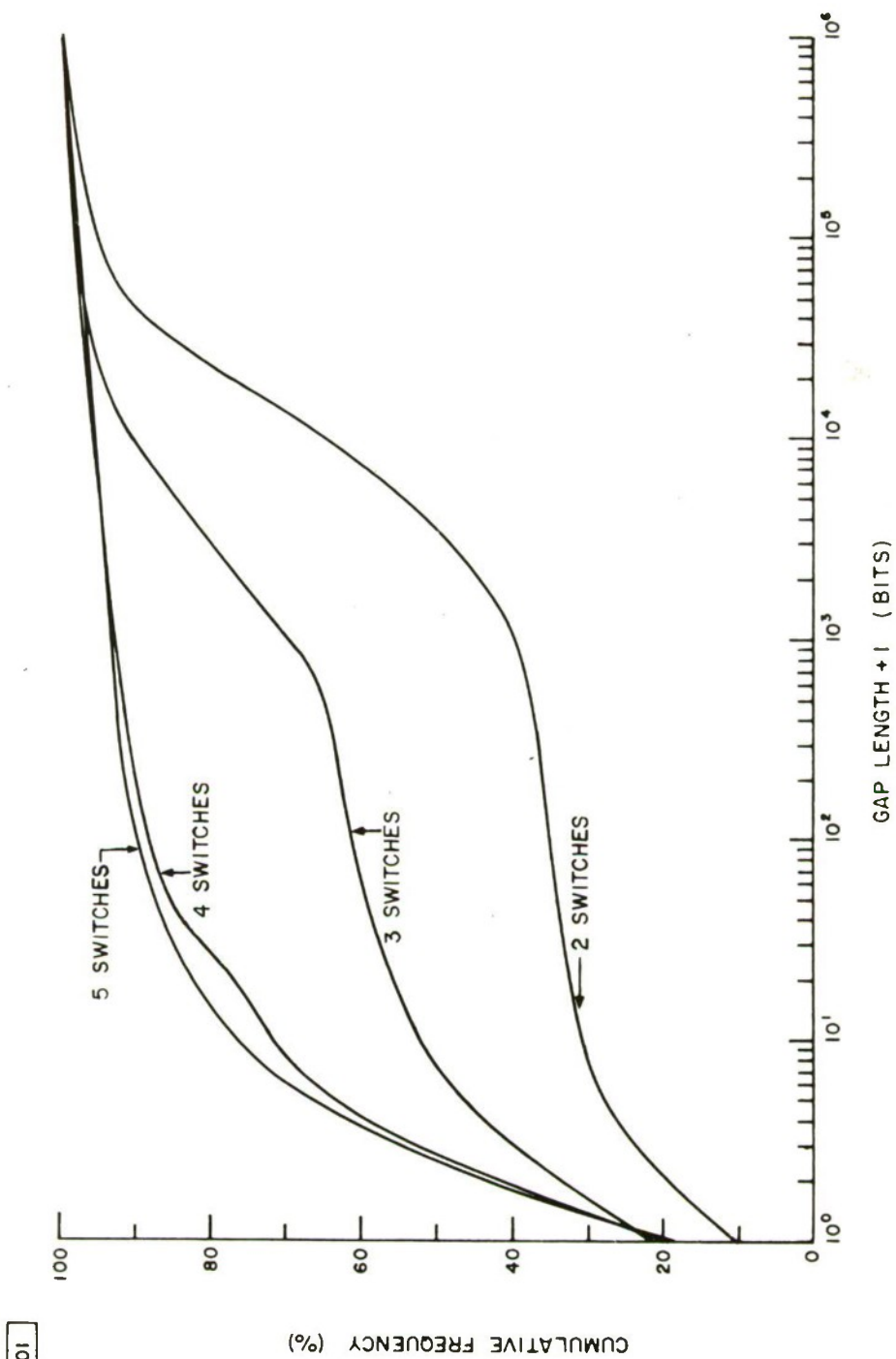


Figure 6. CUMULATIVE DISTRIBUTION OF ERRORFREE GAPS — 4800 b/s

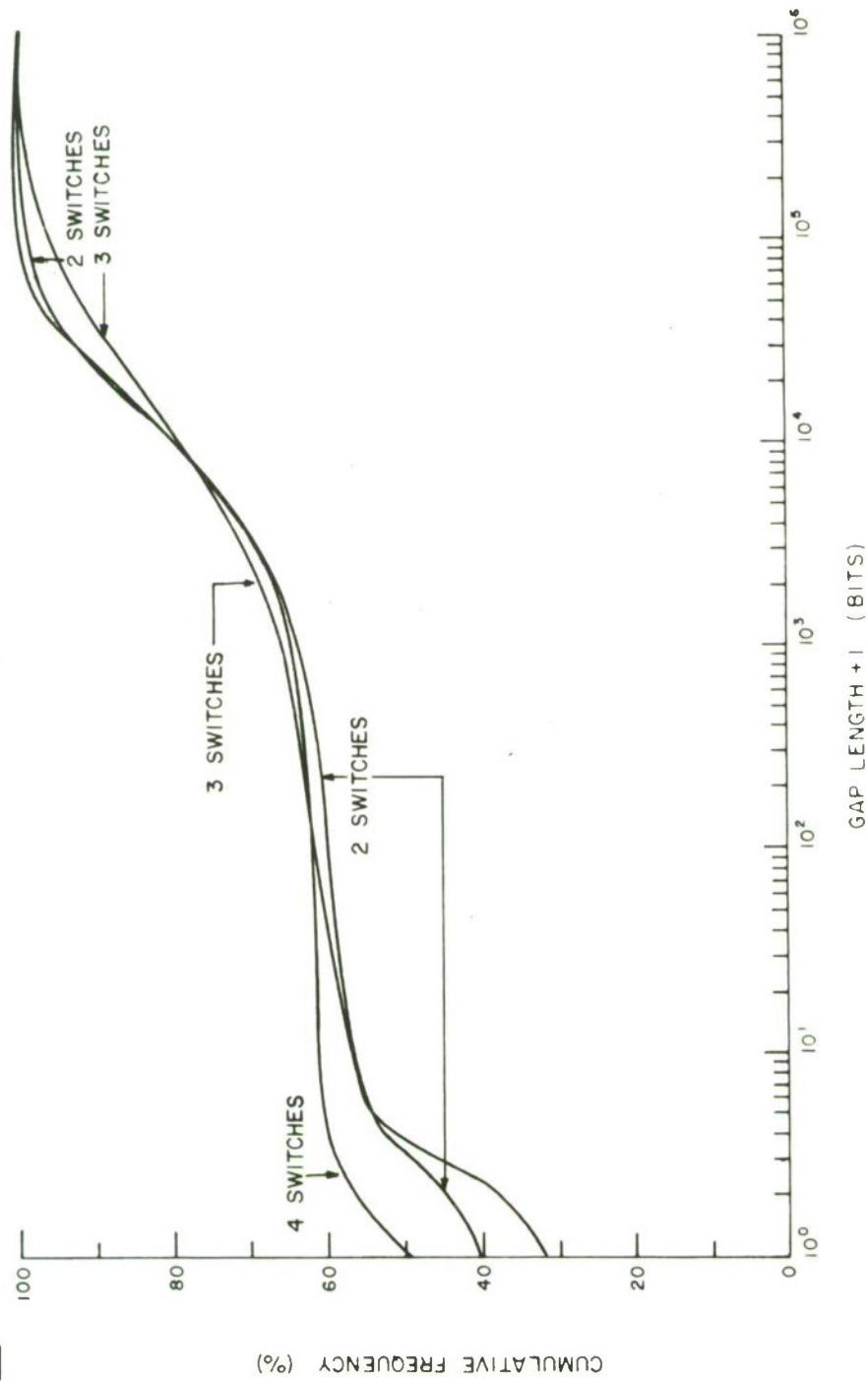


Figure 7. CUMULATIVE DISTRIBUTION OF ERRORFREE GAPS — 9600 b/s



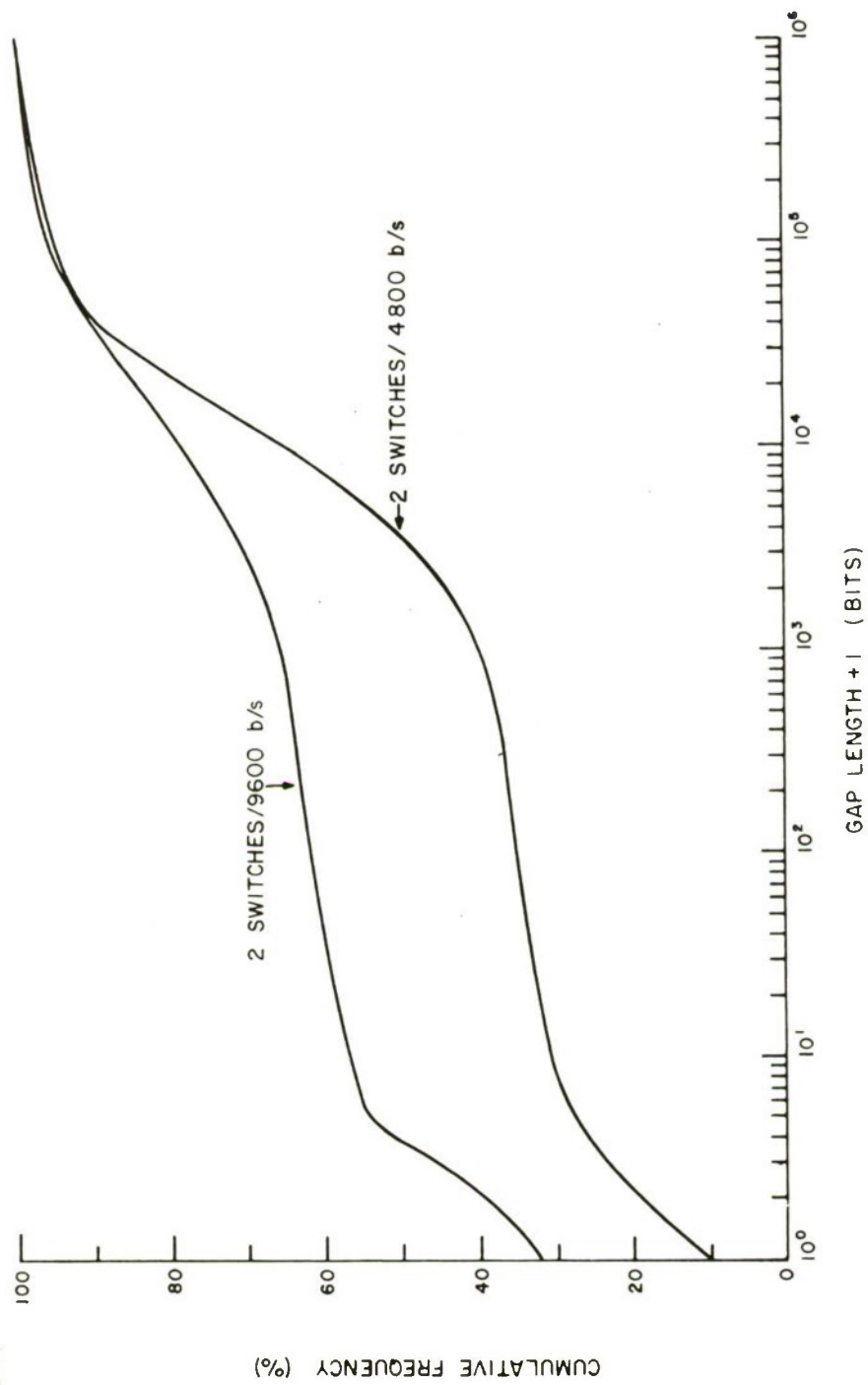


Figure 8. CUMULATIVE DISTRIBUTION OF ERRORFREE GAPS — 4800 vs 9600 b/s, 2 SWITCHES

1A-42,803

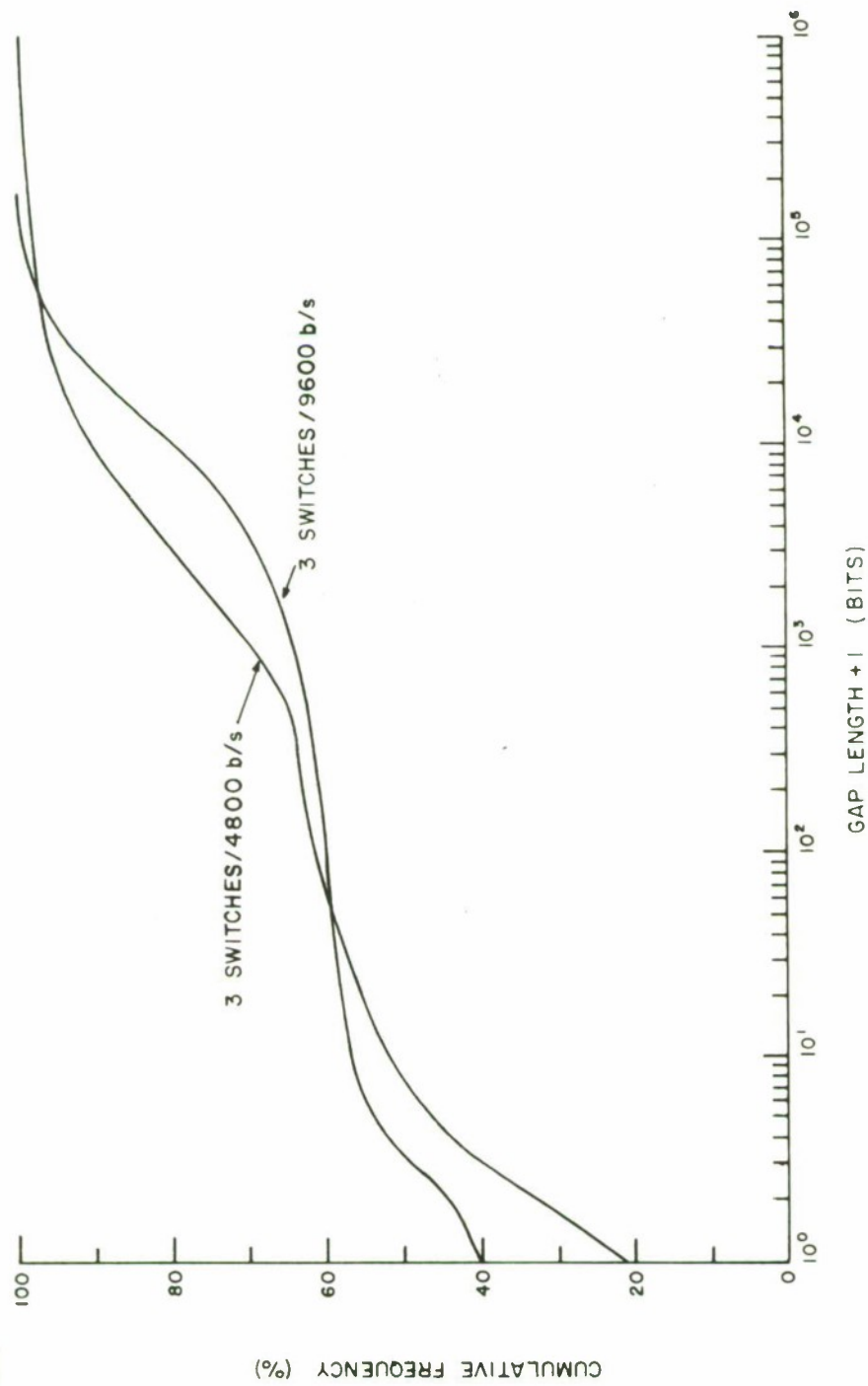


Figure 9. CUMULATIVE DISTRIBUTION OF ERRORFREE GAPS — 4800 vs 9600 b/s, 3 SWITCHES

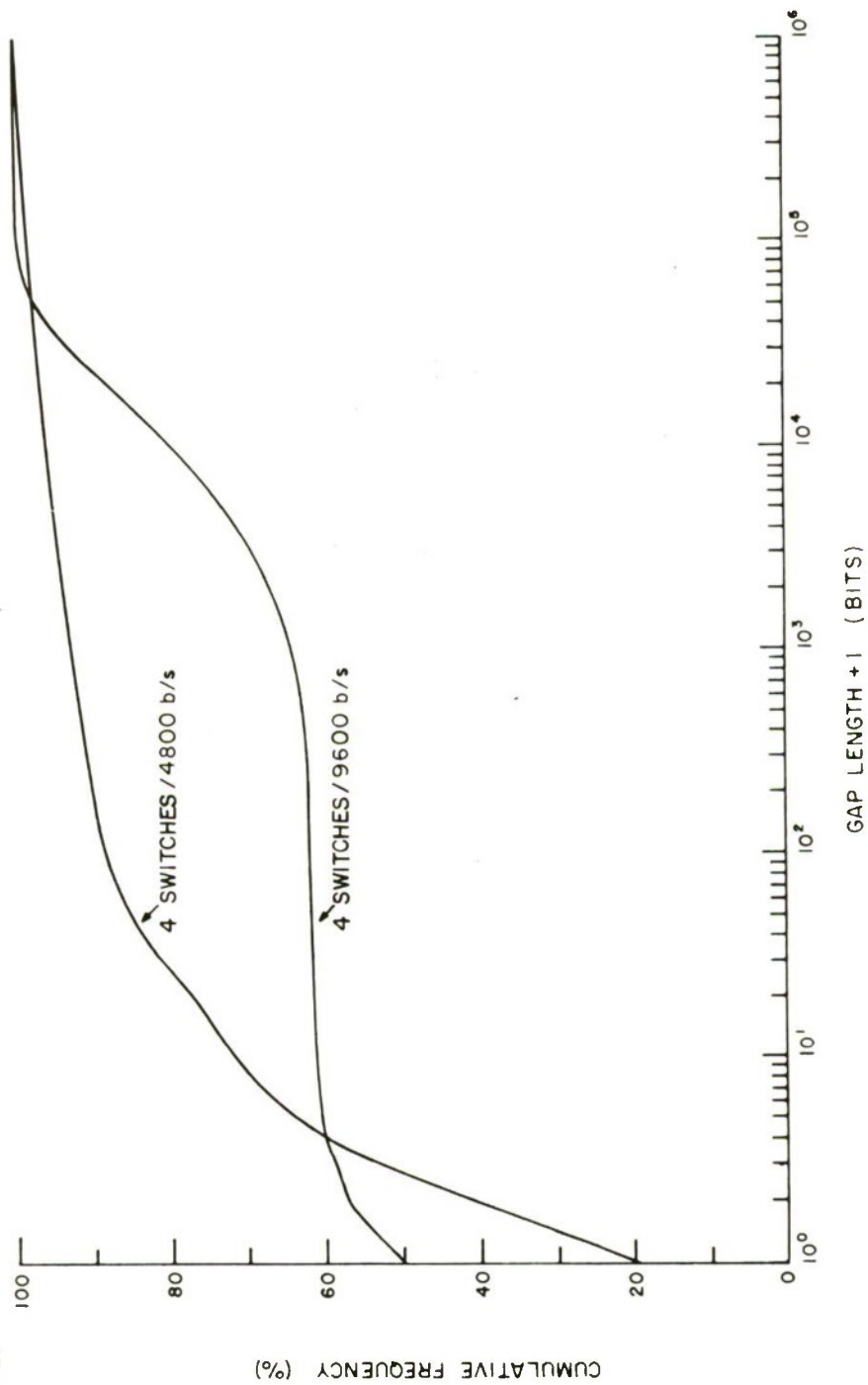


Figure 10. CUMULATIVE DISTRIBUTION OF ERRORFREE GAPS — 4800 vs 9600 b/s, 4 SWITCHES

The explanation, of course, becomes obvious when one realizes that part of the transmission plant is via microwave link which is well known to be a fading channel exhibiting relatively short fades and a time diffuse burst error pattern. This exhibition of diffuse bursts is more prevalent in the 9600 b/s data than in the 4800 b/s data.

#### Block Error Probabilities

The probability that a message block will have at least one error and therefore be an "errored block" is presented in Figures 11 through 15. Within either the 4800 b/s or the 9600 b/s data the probabilities are substantially the same, independent of the number of selected switches in tandem. This is, of course, a comforting fact since whether the probability is high or low at least a practical communication system will not experience varying rates of errored blocks depending upon the complexity of the transmission path. At 4800 b/s the block error probabilities range from  $2 \times 10^{-4}$  to  $8 \times 10^{-2}$  while at 9600 b/s this range is from  $2 \times 10^{-4}$  to  $2 \times 10^{-1}$ . For a typical block length in the order of 2000 bits the block error probability is in the neighborhood of .025 at 4800 b/s but rises to .1 at 9600 b/s.

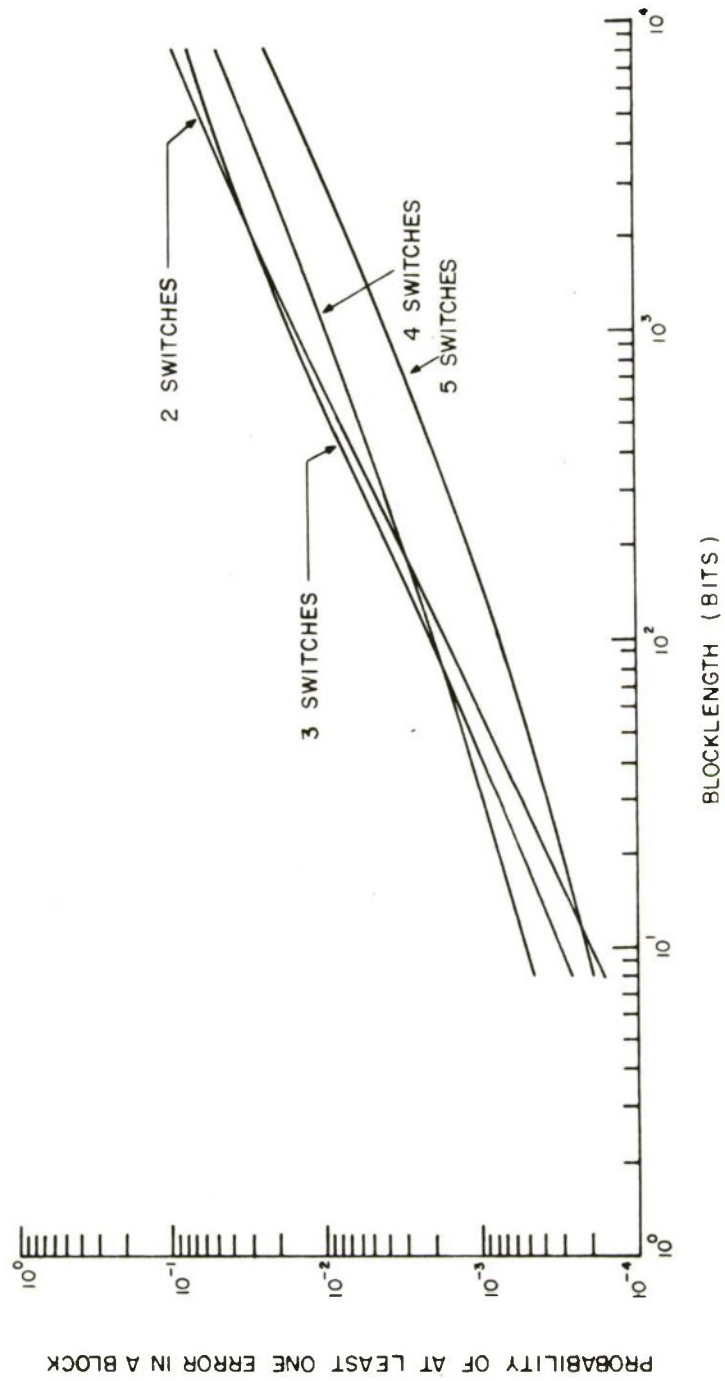


Figure II. BLOCK ERROR RATES — 4800 b/s



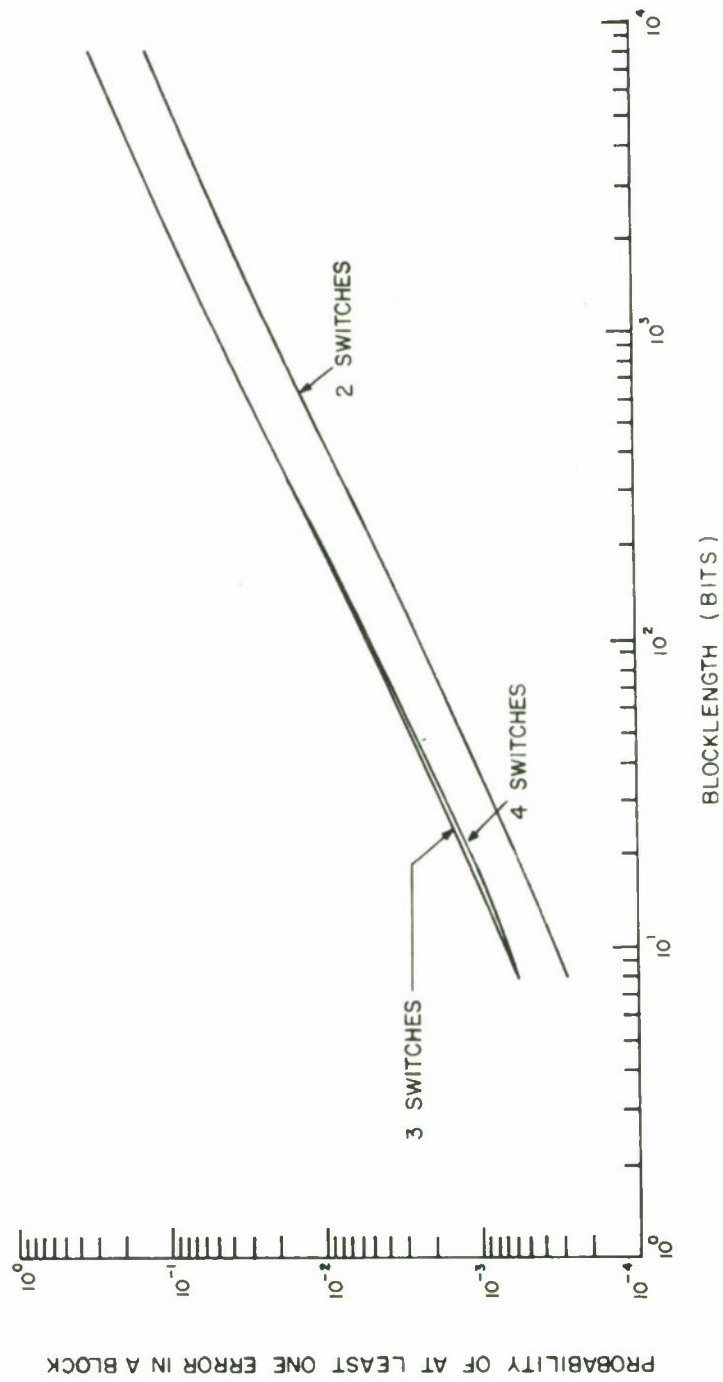


Figure 12. BLOCK ERROR RATES — 9600 b/s

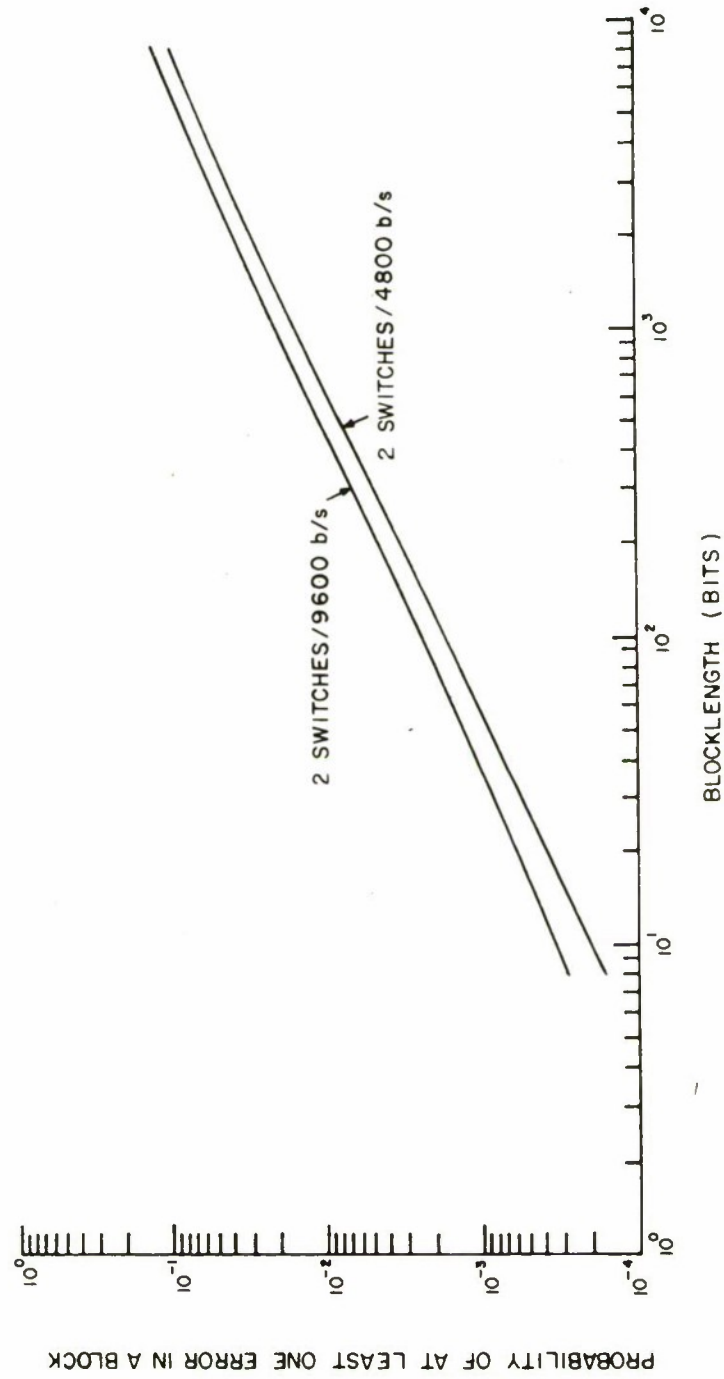


Figure 13. BLOCK ERROR RATES — 4800 vs 9600 b/s, 2 SWITCHES

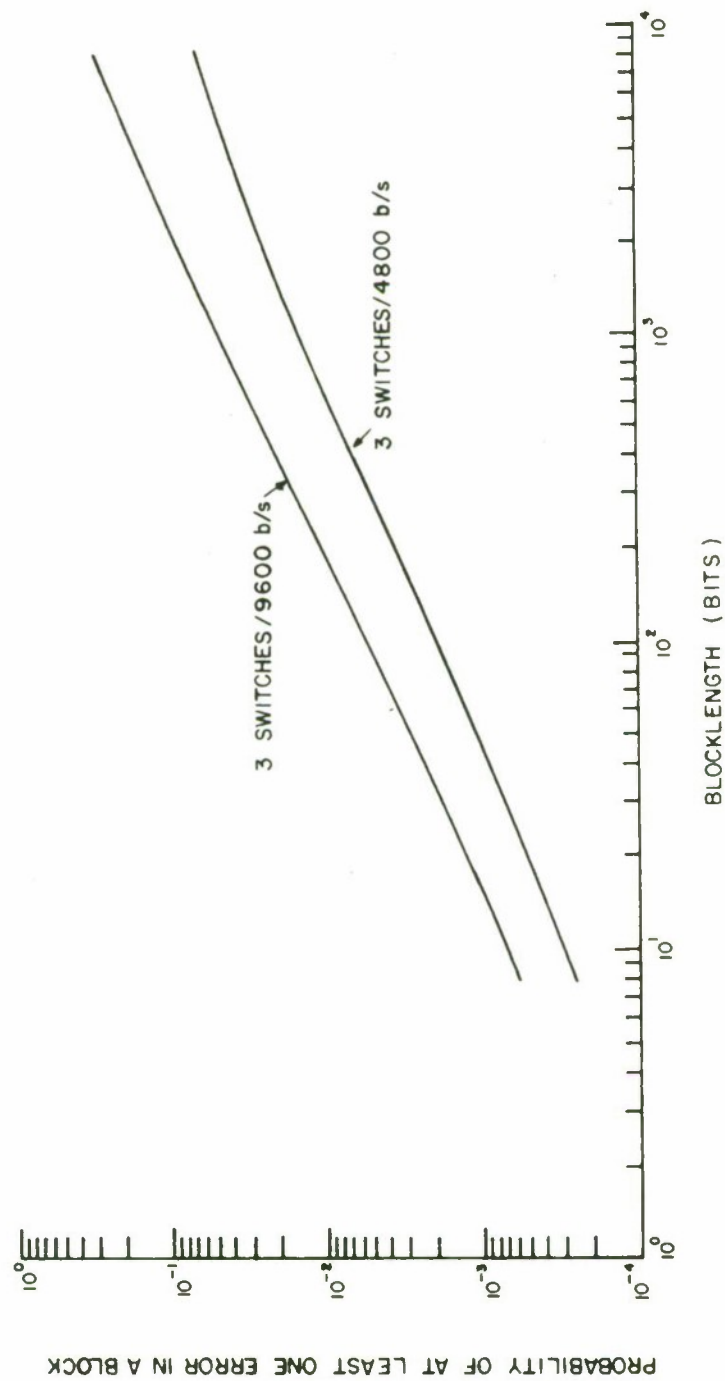


Figure 14. BLOCK ERROR RATES — 4800 vs 9600 b/s, 3 SWITCHES

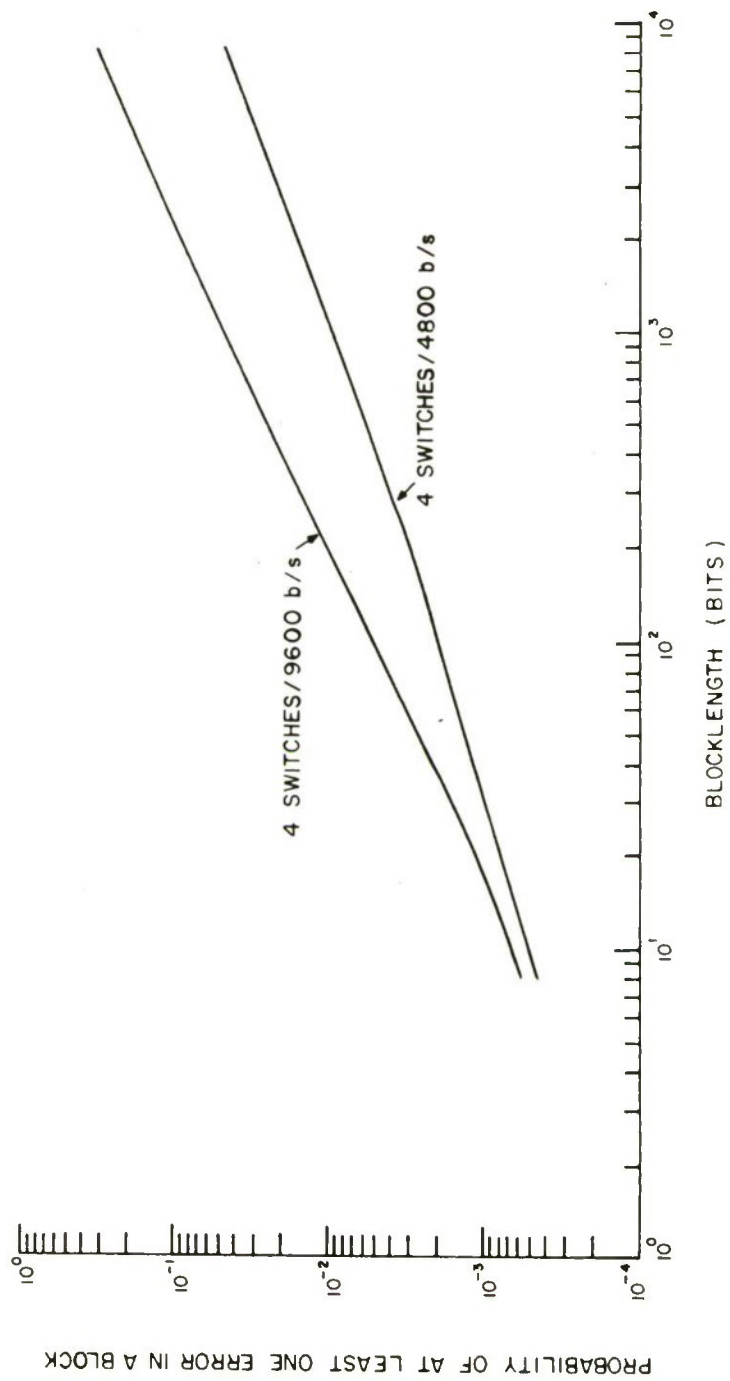


Figure 15. BLOCK ERROR RATES - 4800 vs 9600 b/s, 4 SWITCHES

### SECTION III

#### MARKOV CHANNEL MODEL

The channel data collected is very useful for evaluating the performance of various techniques for error detection and correction that might be applied to the channel. Unfortunately the channel data is most useful to those that have access to it along with large amounts of computer time and sophisticated computer programs. It is, however, possible to calculate the performance of such techniques if a channel model is available. One such model is the MARKOV chain model.

##### The MARKOV Chain Model

Consider the output  $y_k$  of the digital communication channel to result from the sum of the input  $x_k$  and the noise  $e_k$ . Assume that the noise is independent of the input  $x_k$ . Since the noise and in turn the error sequence is assumed to be independent of the input sequence the channel may be completely characterized by its error sequence  $E=(e_k: k=1, 2, \dots)$ . The error sequence is a bit stream of 0's and 1's where an error bit is represented by a 1, and an error-free bit is represented by a 0.

A distribution function which can be calculated for such an error sequence is the error-free run distribution,  $P(0^m|1)$ . This distribution is the conditional probability that, given an error has occurred, it will be followed by at least  $J$  consecutive error-free bits.



The error-free run distribution which is a close relative of the error-free gap distribution is used to obtain the channel model. In obtaining the model it is further assumed that the error sequence has a limited number of states and the probability of being in any particular state at the  $n$ th bit decision is conditionally dependent only upon the state during the  $(n-1)$ th bit decision. Such a process is called a MARKOV process of order one and can be represented by an  $N$  state MARKOV chain.

Fritchman (2) has developed a model for an  $N$  state MARKOV chain which partitions the  $N$  states into two groups of states,  $A$  and  $B$ . The  $K$  ( $K < N$ ) states in group  $A$  correspond to  $K$  states where errors cannot occur. The  $N-K$  states in group  $B$  correspond to the states in which errors can occur. In order to simplify the mathematics Fritchman made two restrictions on the model. First he did not allow transitions among the error states or among the error-free states. Second, he limited the model to a single error state,  $K = N-1$ . The state transition matrix for the MARKOV chain then becomes:

$$P = \begin{bmatrix} p_{11} & & & & & p_{1N} \\ & & 0 & & & \\ & p_{22} & & & & p_{2N} \\ & & \cdot & & & \cdot \\ 0 & & \cdot & & & \cdot \\ & & & \cdot & & \cdot \\ & & & & p_{N-1, N-1} & \\ p_{N1} & p_{N2} & \cdots & p_{N, N-1} & & p_{N, N} \end{bmatrix}$$

where  $p_{ij}$  is the probability of transition from state  $S_i$  to state  $S_j$ .

Fritchman has shown that, for this model, the  $p_{ij}$  can be uniquely

determined from the error-free run distribution. For a

stochastic matrix,  $\sum_{j=1}^N p_{ij} = 1, i = 1, 2, \dots, N$ , there are only

$2(N-1) = 2K$  unknowns. By fitting a sum of  $K$  exponentials to the error-

free run distribution, the  $2K$  unknowns may be determined. If the

error-free run distribution can be approximated by

$$P(0^m | 1) = A_1 e^{a_1 m} + A_2 e^{a_2 m} + \dots + A_K e^{a_K m}$$

then, Fritchman has shown that the transition matrix is given by

$$\begin{bmatrix} e^{a_1} & & & & & 1 - e^{a_1} \\ & e^{a_2} & & 0 & & 1 - e^{a_2} \\ & & \ddots & & & \vdots \\ & & & \ddots & & \vdots \\ & & & & e^{a_K} & 1 - e^{a_K} \\ 0 & & & & & \vdots \\ A_1 e^{a_1} & A_2 e^{a_2} & \dots & A_K e^{a_K} & & 1 - \sum_{j=1}^K A_j e^{a_j} \end{bmatrix}$$

The matrix is determined by applying a computer program to determine

the values of the  $A$ 's and  $a$ 's that best fit the data error-free run

distribution. The program starts by assuming a two state model ( $K=1$ )

and increments K until a fit to the data error-free run distribution is achieved by the exponential polynomial expression.

#### Data Derived Channel Models

The error-free run distributions (derived from the gap distributions) for the seven selected configurations of switches were fitted by sums of exponentials and the state transition matrices were calculated. The results are presented in Tables 3 through 9. Since there is only one error state,  $K=N-1$ , the conditional probability of error is the probability  $p_N$  of being in state N, and an average bit error probability (3) is found to be given by

$$p_e = \left[ 1 + \sum_{j=1}^{N-1} \left( \frac{p_{Nj}}{p_{jN}} \right) \right]^{-1}$$

For each of the transition matrices, the bit-error probability is given for the model, and the goodness of fit of the model is reported. In all cases the RMS error between the data error-free run distribution and the model predicted distribution is less than .15. In all cases the model was validated to the level of RMS error by generating pseudo-error pattern data from it and performing an evaluation of comparative gap distribution functions.

TABLE 3  
MARKOV CHAIN MODEL - TWO SELECTED SWITCHES - 4800 b/s

$$P = \begin{bmatrix} 0.9754047 & 0.0 & 0.0 & 0.0245953 \\ 0.0 & 0.9995566 & 0.0 & 0.0004434 \\ 0.0 & 0.0 & 0.9999969 & 0.0000031 \\ 0.5131625 & 0.2505878 & 0.0895789 & 0.1466708 \end{bmatrix}$$

$$p_e = 3.39 \times 10^{-5}, \text{ interdistribution RMS error} = .145$$

TABLE 4

MARKOV CHAIN MODEL - THREE SELECTED SWITCHES - 4800 b/s

$$P = \begin{bmatrix} 0.2156599 & 0.0 & 0.0 & 0.0 & 0.0 & 0.7843401 \\ 0.0 & 0.8886233 & 0.0 & 0.0 & 0.0 & 0.1113767 \\ 0.0 & 0.0 & 0.9987018 & 0.0 & 0.0 & 0.0012982 \\ 0.0 & 0.0 & 0.0 & 0.9999393 & 0.0 & 0.0000607 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.9999977 & 0.0000023 \\ 0.1124190 & 0.1780878 & 0.1994085 & 0.2057504 & 0.0209361 & 0.2833981 \end{bmatrix}$$

$$p_e = 7.93 \times 10^{-5}, \text{ interdistribution RMS error} = .019$$

TABLE 5

MARKOV CHAIN MODEL - FOUR SELECTED SWITCHES - 4800 b/s

$$P = \begin{bmatrix} 0.9611693 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0388307 \\ 0.0 & 0.8898716 & 0.0 & 0.0 & 0.0 & 0.1101284 \\ 0.0 & 0.0 & 0.9988276 & 0.0 & 0.0 & 0.0011724 \\ 0.0 & 0.0 & 0.0 & 0.9999507 & 0.0 & 0.0000483 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.9999969 & 0.0000031 \\ 0.2044374 & 0.2271502 & 0.0585721 & 0.0422230 & 0.0166381 & 0.4509793 \end{bmatrix}$$

$$p_e = 1.58 \times 10^{-4}, \text{ interdistribution RMS error} = .047$$

TABLE 6

MARKOV CHAIN MODEL - FIVE SELECTED SWITCHES - 4800 b/s

$$P = \begin{bmatrix} 0.9068574 & 0.0 & 0.0 & 0.0 & 0.0931426 \\ 0.0 & 0.9980199 & 0.0 & 0.0 & 0.0019801 \\ 0.0 & 0.0 & 0.9999507 & 0.0 & 0.0000493 \\ 0.0 & 0.0 & 0.0 & 0.9999981 & 0.0000019 \\ 0.3606889 & 0.0668155 & 0.0379466 & 0.0305986 & 0.5039504 \end{bmatrix}$$

$$p_e = 6.05 \times 10^{-5}, \text{ interdistribution RMS error} = .054$$

TABLE 7

MARKOV CHAIN MODEL - TWO SELECTED SWITCHES - 9600 b/s

$$P = \begin{bmatrix} 0.9995636 & 0.0 & 0.0004364 \\ 0.0 & 0.9999922 & 0.0000078 \\ 0.4874004 & 0.1026200 & 0.4099796 \end{bmatrix}$$

$$p_e = 7.01 \times 10^{-5}, \text{ interdistribution RMS error} = .139$$



TABLE 8

MARKOV CHAIN MODEL - THREE SELECTED SWITCHES - 9600 b/s

$$P = \begin{bmatrix} 0.9982991 & 0.0 & 0.0017119 \\ 0.0 & 0.9999714 & 0.0000286 \\ 0.3635153 & 0.2268990 & 0.4095857 \end{bmatrix}$$

$$p_e = 1.23 \times 10^{-4}, \text{ interdistribution RMS error} = .126$$

TABLE 9

MARKOV CHAIN MODEL - FOUR SELECTED SWITCHES - 9600 b/s

$$P = \begin{bmatrix} 0.9999391 & 0.0000609 \\ 0.3979892 & 0.6020108 \end{bmatrix}$$

$$p_e = 1.52 \times 10^{-4}, \text{ interdistribution RMS error} = .018$$

## SECTION IV

### BURST ANALYSIS

The previous discussion of error pattern distributions does not present the complete picture. There is no information about length of bursts, nor is there information relative to the interval between bursts (guard space). For this reason the data shall be evaluated in terms of burst distributions.

#### Definition of a Burst (1)

A burst is defined as region of the serial data stream where the following properties hold. A minimum number of errors,  $M_e$ , are contained in the region and the minimum density of errors in the region is  $\Delta$ . Both of these conditions must be satisfied for the chosen values of  $M_e$  and  $\Delta$  for the region to be defined as a burst. The density of errors is defined as the ratio of bits in error to the total number of bits in the region.

The following properties also hold for any burst. The burst always begins with a bit in error and ends with a bit in error. A burst may contain correct bits. Each burst is immediately preceded and followed by an interval in which the density of errors is less than  $\Delta$ .

The burst probability density function is defined as the probability of occurrence of a burst of length  $N$  where  $N$  is any

positive integer. The burst length is equal to the total number of bits in the burst. A separate burst probability density function may be determined for each pair of  $\Delta$  and  $M_e$  values.

The minimum number of errors in a burst has been chosen to be two for all the data included here. Experience indicates that larger values of  $M_e$  would not change the values of burst length significantly. When a value of one is selected for  $M_e$ , every error becomes a burst and the requirement that a burst begin and end in different errors is violated. Consequently, the burst distribution reduces to the consecutive error distribution. While a minimum value

$\Delta$  is used in defining bursts the actual burst error density is calculated and the algorithm that applies the definition to the data has the effect of maximizing this density (since all bursts terminate in an error.)

#### Definition of Interval

The interval is defined as the region of the serial data stream where the following properties hold. The minimum density of errors is less than  $\Delta$ , and the region begins and ends in a correct bit. An interval may contain errors. An interval is always immediately preceded and followed by a burst. Thus, each and every bit in the data stream must lie in either a burst region or an interval region.

The interval probability density function is defined as the probability of occurrence of an interval of length  $L$ , where  $L$  is

any positive integer. The interval probability density is a joint function of both  $\Delta$  and  $M_e$ . Use of  $M_e = 1$  has the effect of reducing the interval distribution to the error-free gap distribution.

The guard space ratio is defined as the ratio of an interval length to the burst length preceding it.

#### Burst Distributions

The distribution functions for burst lengths and burst densities<sup>\*</sup> are presented on Figures 16 through 17 for the 4800 b/s data and on Figures 18 through 19 for the 9600 b/s. At 4800 b/s significant percentages of the bursts ( > 15%) are longer than 100 bits and the majority of the bursts are more than 30% dense in errors. The 9600 b/s data bursts tend to be shorter than their 4800 b/s counterparts but are of comparable density to their 4800 b/s counterparts. The burst distributions tend to imply that a greater proportion of microwave fading links than wireline links were obtained when the calls were dialed at random.

#### Interval Distributions

For both the 4800 b/s data (Figures 20 through 21) and the 9600 b/s data (Figures 22 through 23) the intervals between bursts are long and for the most part error-free. Here again, as in the

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\*The burst density criteria  $\Delta$ , was chosen as 0.05 since, for a variation of approximately .1 above .05 the computer results remain the same and indicate the results have independence of the definition.

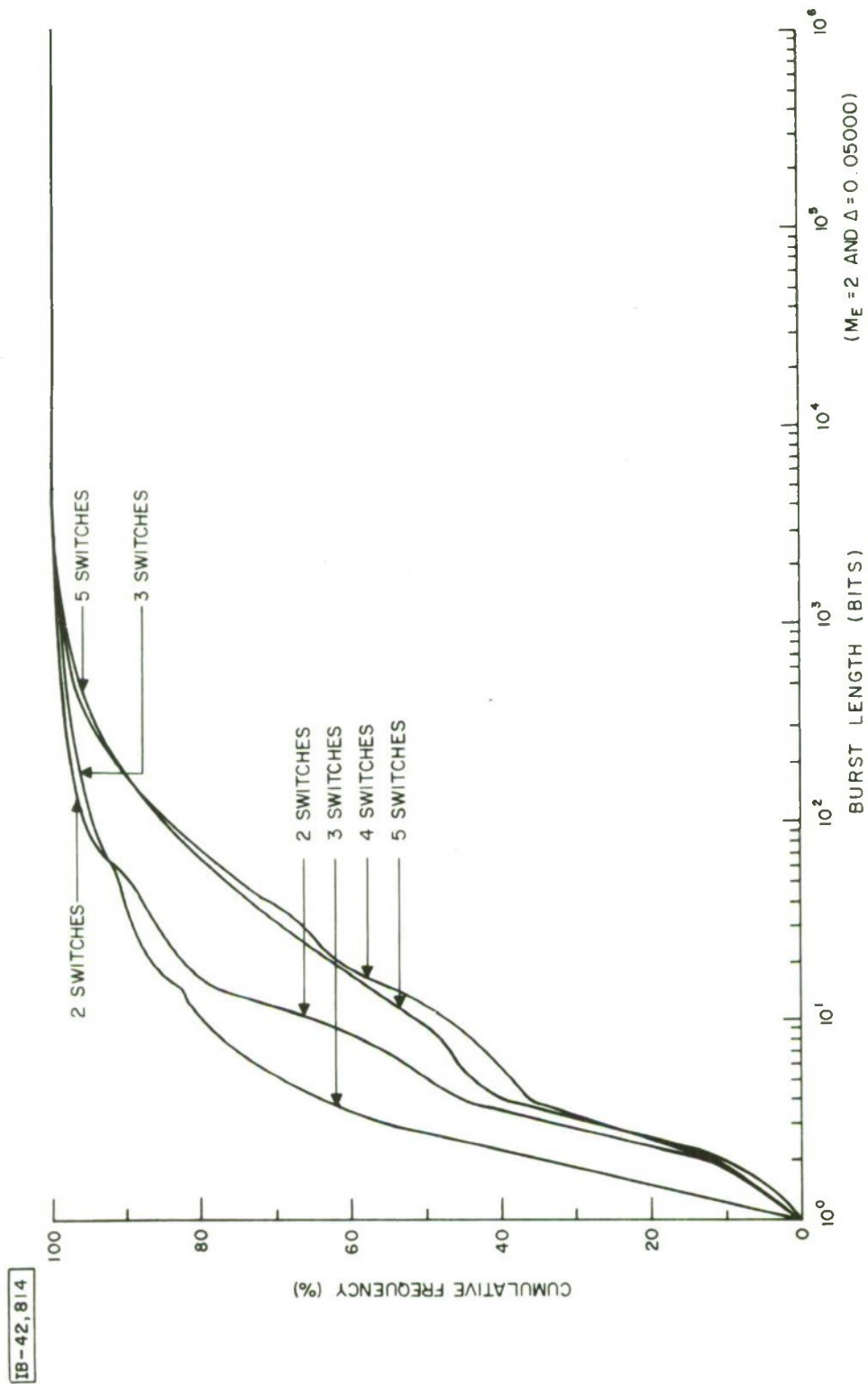


Figure 16. DISTRIBUTION OF BURSTS — 4800 b/s

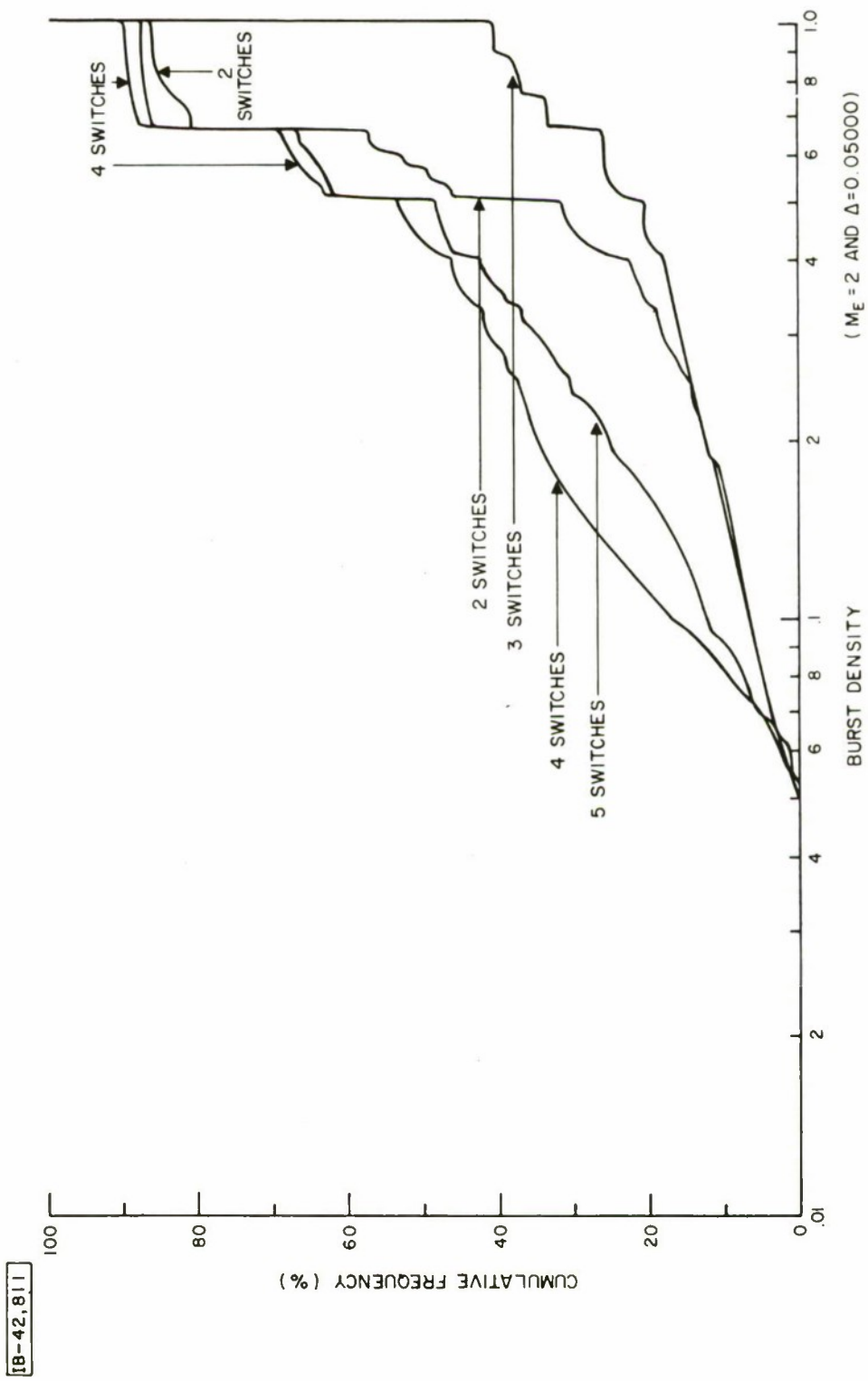


Figure 17. DISTRIBUTION OF BURST DENSITIES — 4800 b/s



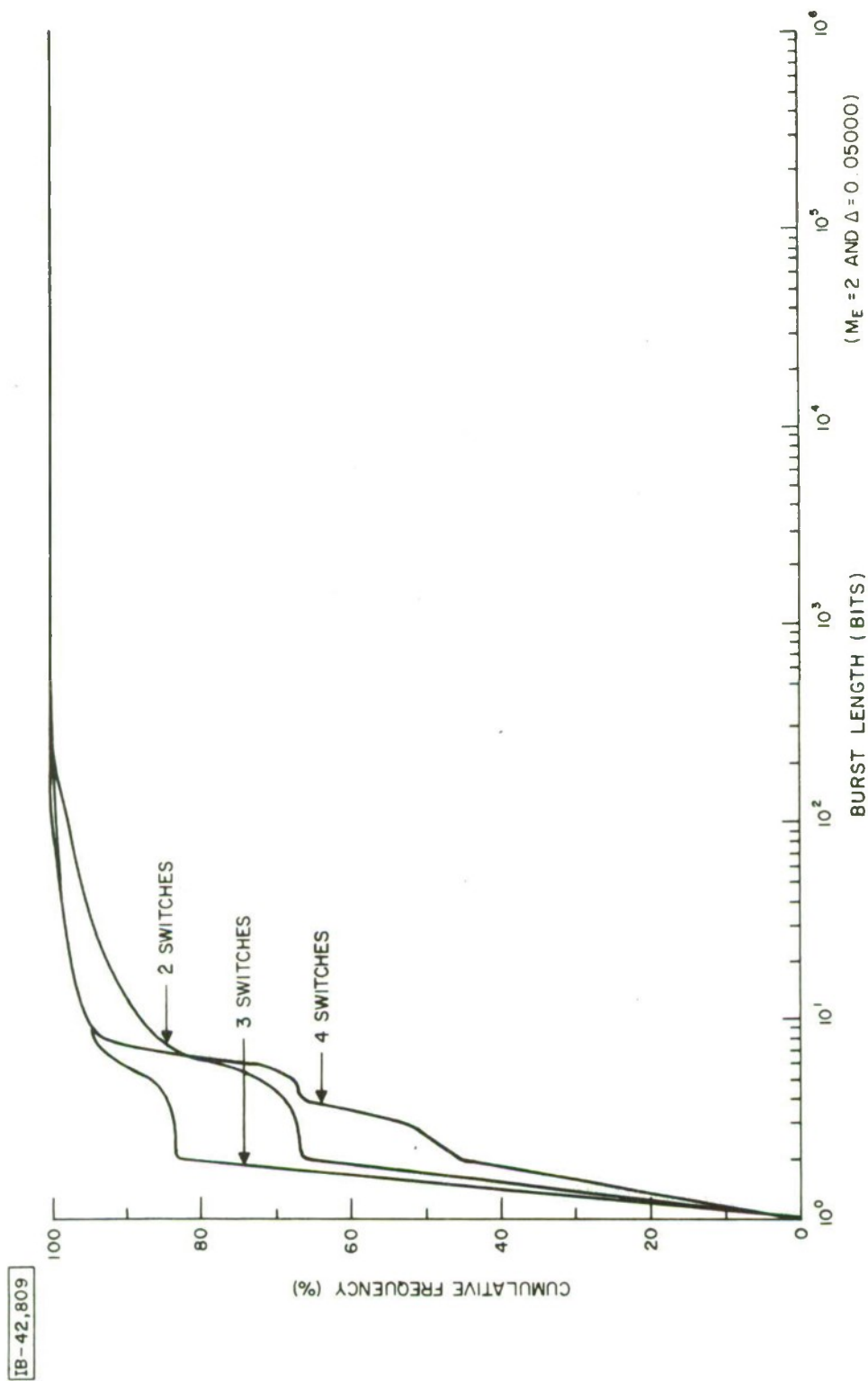


Figure 18. DISTRIBUTION OF BURSTS - 9600 b/s

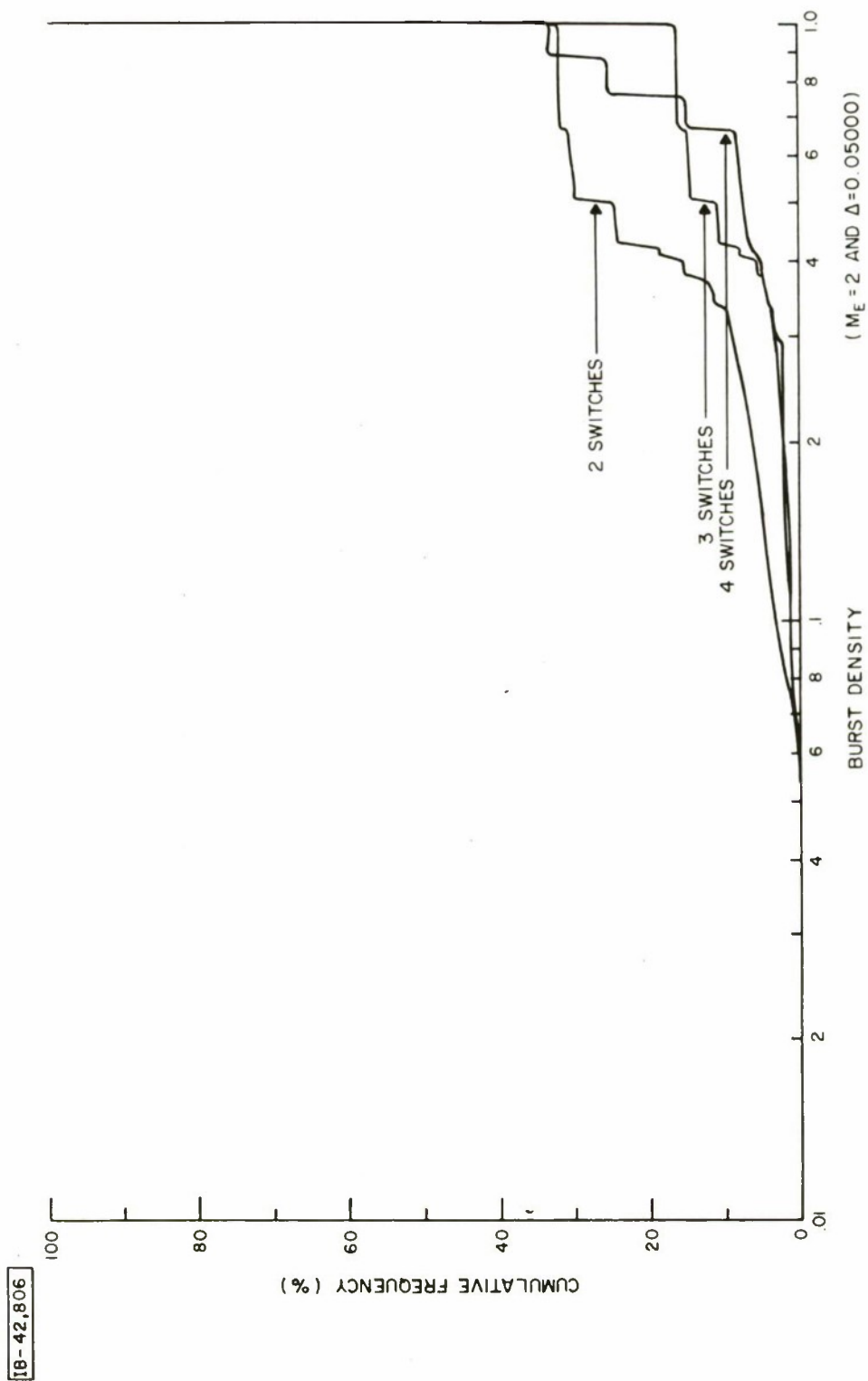


Figure 19. DISTRIBUTION OF BURST DENSITIES — 9600 b/s

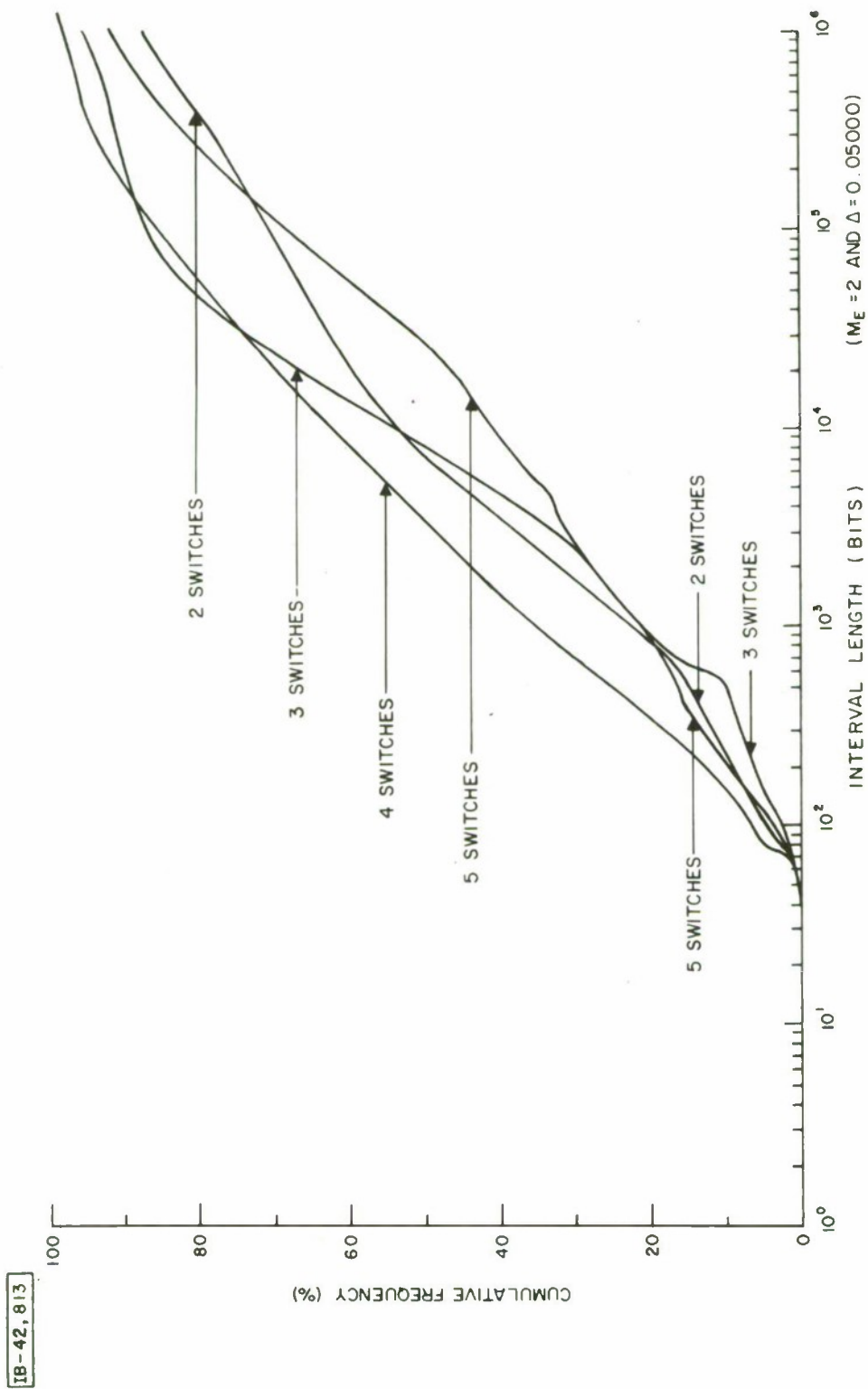


Figure 20. DISTRIBUTION OF INTER-BURST INTERVALS — 4800 b/s

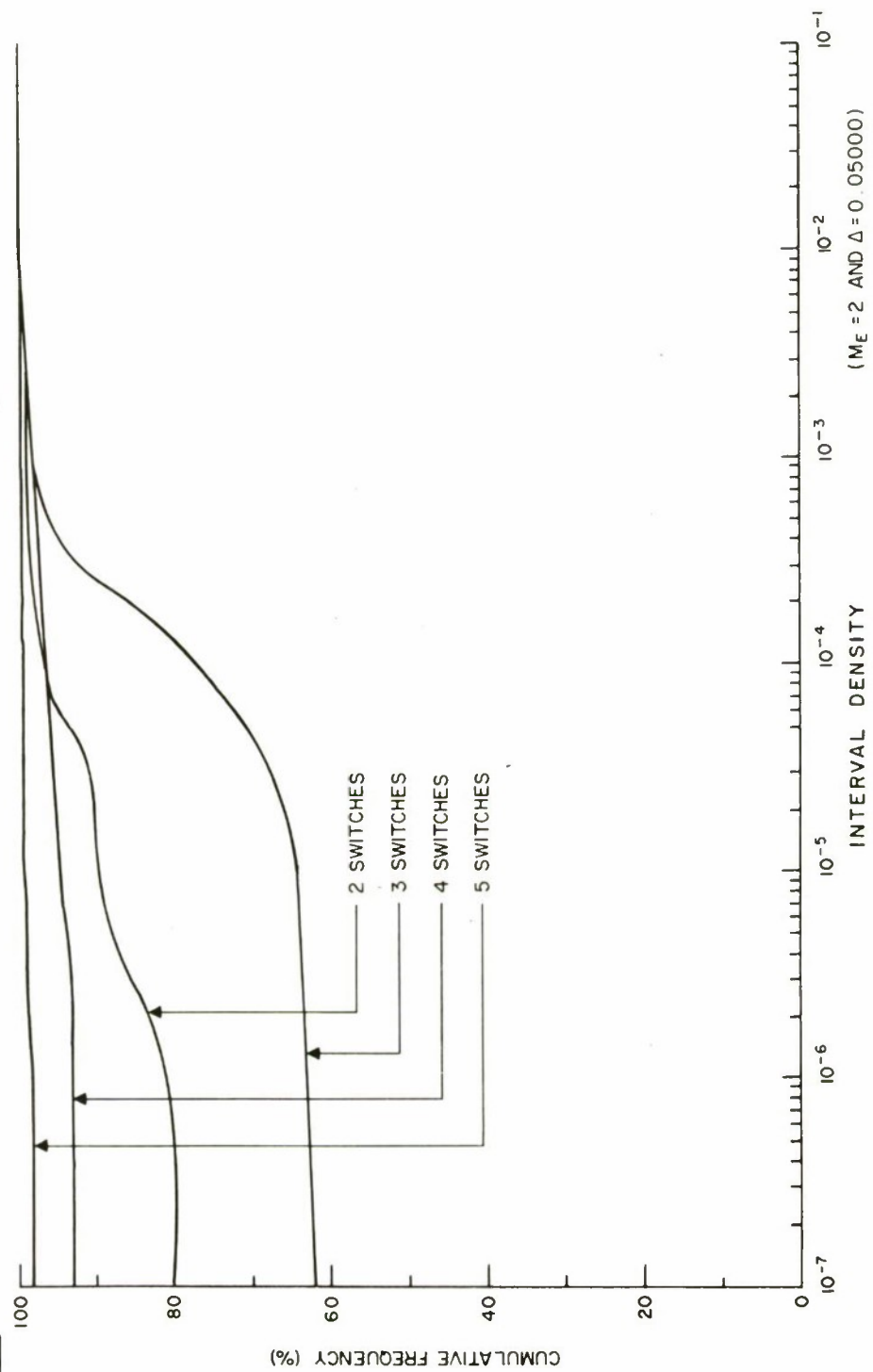


Figure 21. DISTRIBUTION OF INTERVAL-ERROR DENSITIES — 4800 b/s

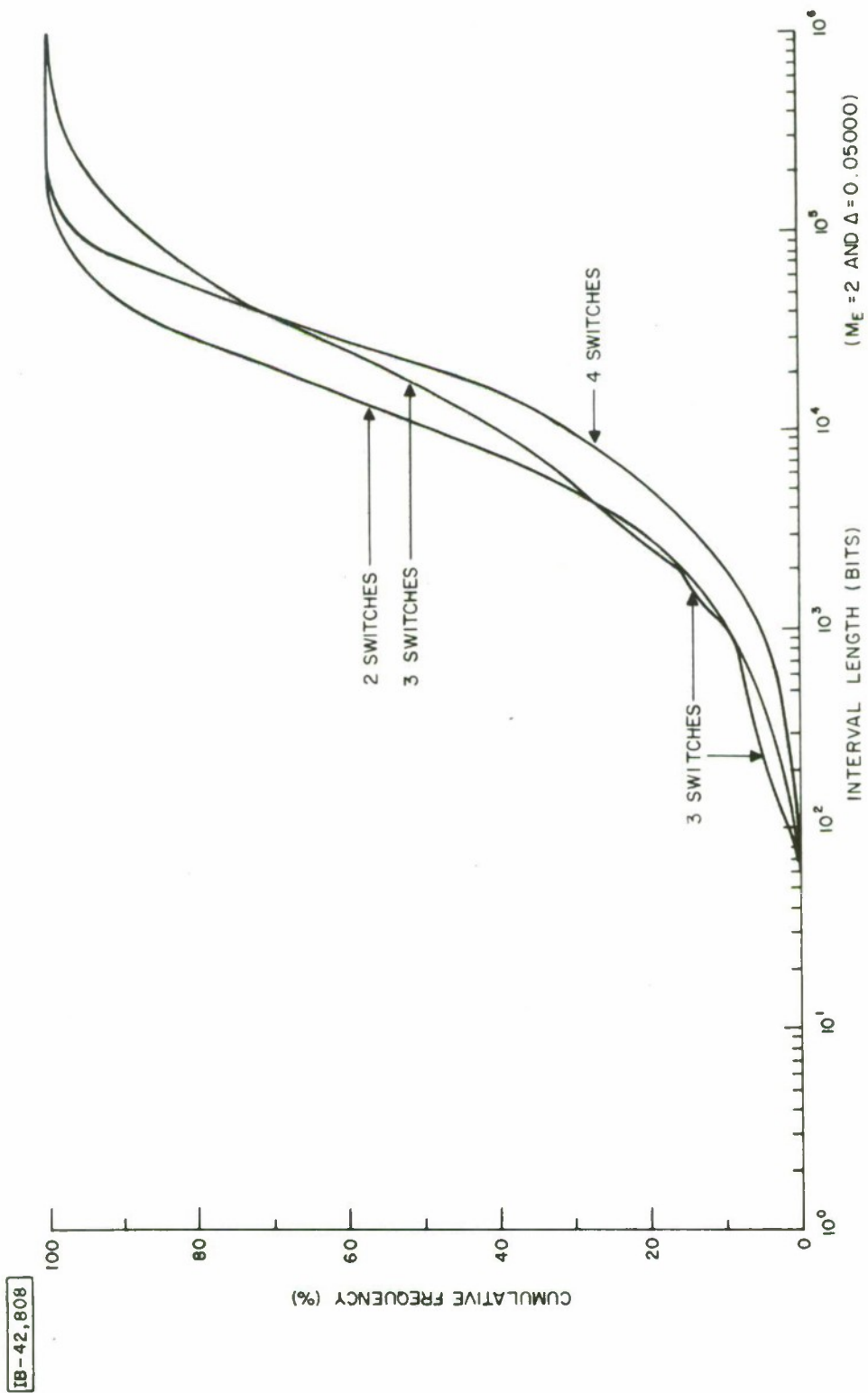


Figure 22. DISTRIBUTION OF INTER-BURST INTERVALS — 9600 b/s

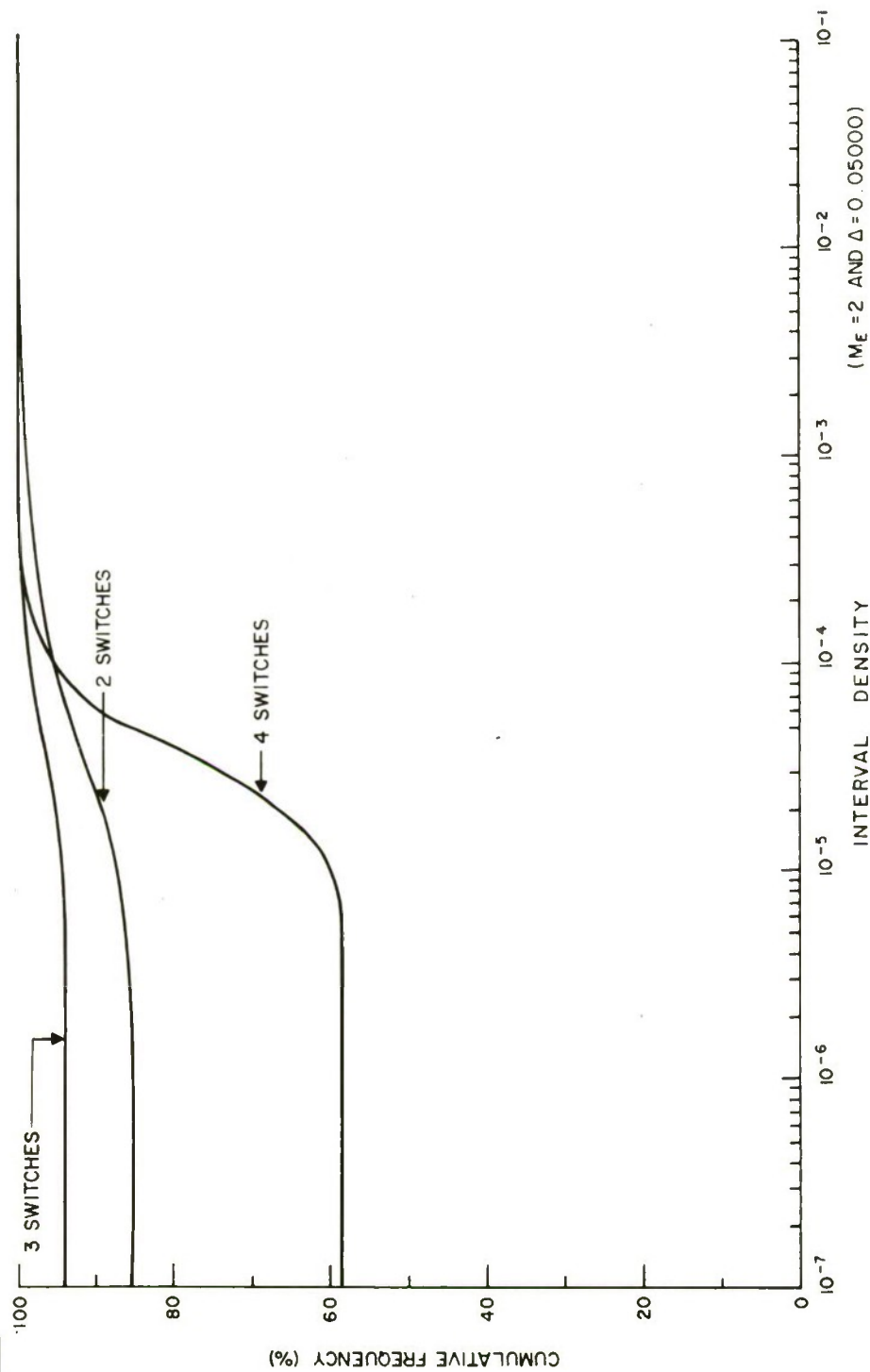


Figure 23. DISTRIBUTION OF INTERVAL-ERROR DENSITIES — 9600 b/s



case of block error rates there is little variation in either burst (or interval) lengths (or densities) with the number of switches taken in tandem. Thus, in terms of switch tandem statistics the channel appears very stable.

#### Guard Space Distribution

While the burst length and interval length distributions show short to intermediate bursts, with a few long bursts, and long to very long intervals, the correlation of the bursts and intervals is important. This distribution commonly called guard space indicates the amount of interval which follows each burst and can be used as an aid in the design of error control systems which depend on long intervals separating error bursts. The guard space distribution for the 4800 and 9600 b/s data is shown in Figures 24 through 25. All but a few (3%) of the bursts are followed by intervals significantly longer than the burst.

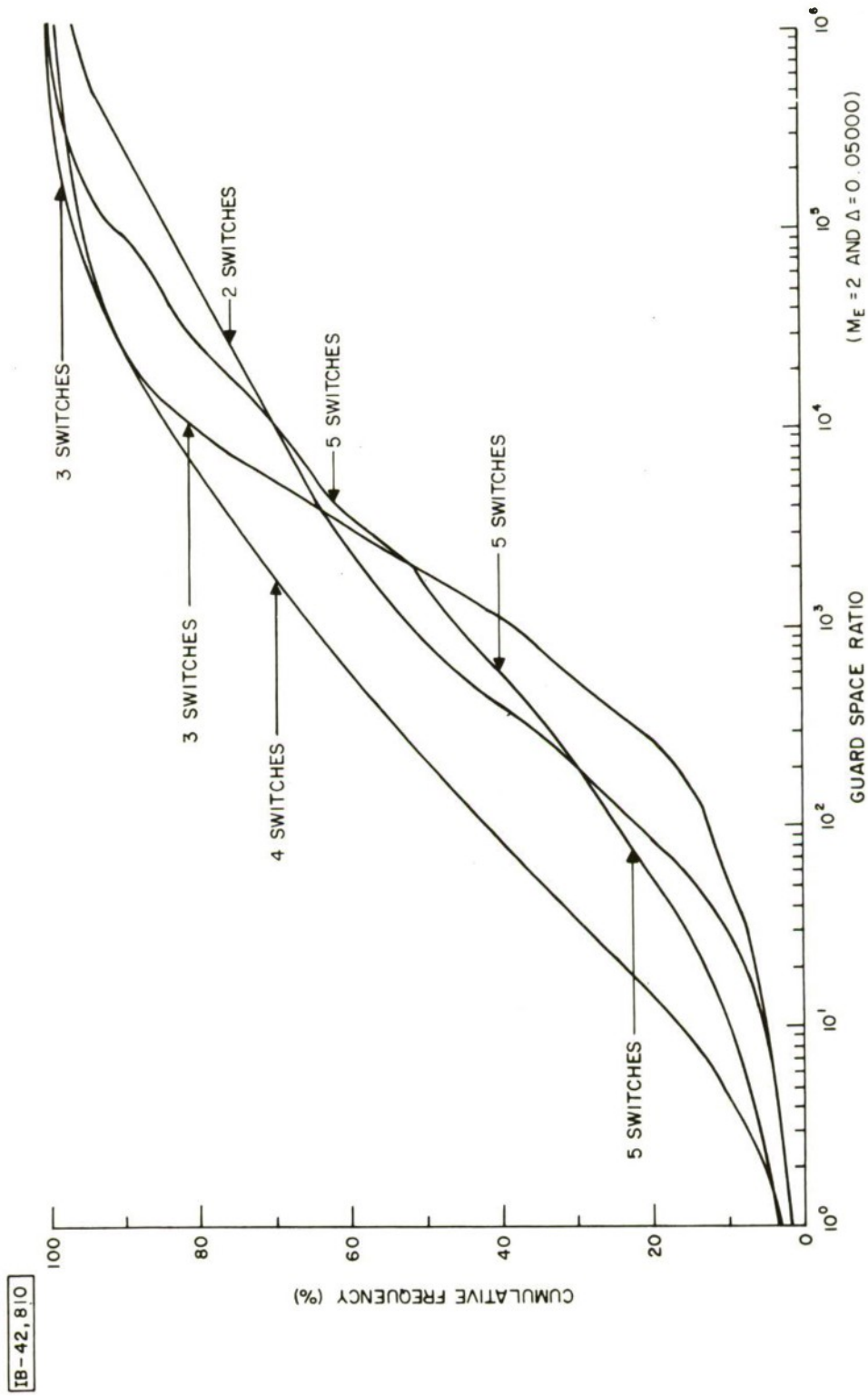


Figure 24. DISTRIBUTION OF BURST GUARD SPACES — 4800 b/s

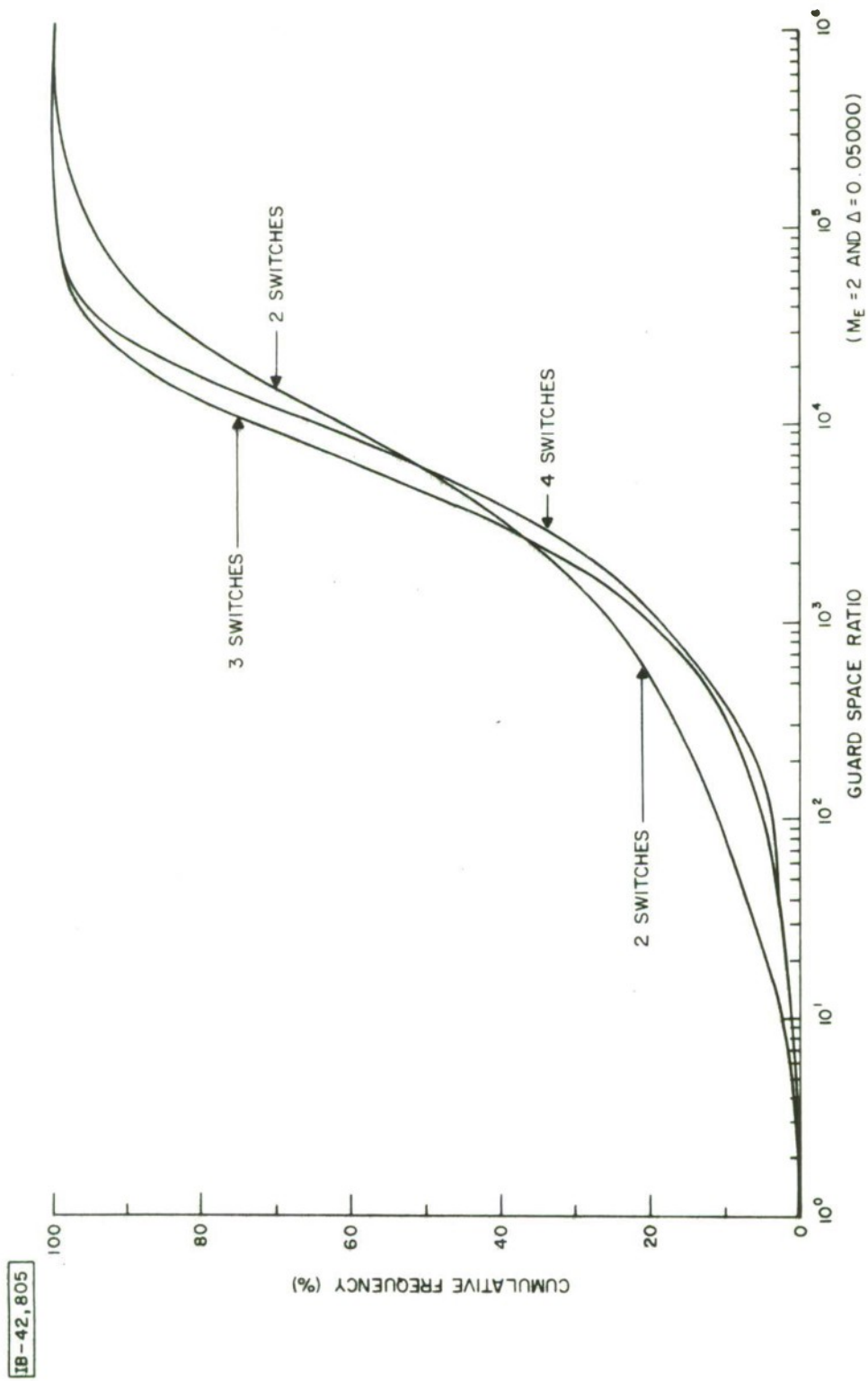


Figure 25. DISTRIBUTION OF BURST GUARD SPACES — 9600 b/s

## SECTION V

### SUMMARY

An interim analysis of error pattern data collected at 4800 b/s and 9600 b/s via digital data transmission on AUTOVON using the Codex 9600 modem has been performed. The data tends to show that errors occur in dense bursts, ranging in length to thousands of bits with significant numbers of bursts of a few hundred bits length. The bursts are generally separated by long error-free intervals. The differences between the 4800 b/s and 9600 b/s data appear to be minimal with fewer bursts at 9600 b/s.

No conclusions should be drawn in terms of a 4800 vs 9600 b/s comparison since the data considered in this interim report was not balanced in terms of numbers of bits collected at the different data rates or the different selected switch connections. When equal amounts of data are available for the various switch configurations it should then be possible to compare 4800 and 9600 b/s data.

It has been demonstrated that an analytical channel model can be fit to the data, namely a MARKOV channel model. This model can be used for coding analysis by those who do not have the raw channel data and data analysis programs.

## REFERENCES

1. Brayer, K., "Error Patterns Measured on Transequatorial HF Communication Links," IEEE Trans. on Communication Technology, April 1968.
2. Fritchman, B. D., A Binary Channel Characterization Using Partitioned Markov Chains with Applications to Error Correcting Codes, Lehigh University, Bethlehem, Pa., June 1967.